Recursive Distinctioning

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Abstract
In this paper we explore Recursive Distinctioning.

Keywords: Recursive Distinctioning, algebra, topology, biology, replication, cellular automaton, quantum, DNA, container, extainer

1. Introduction to Recursive Distinctioning

Recursive Distinctioning (RD) is a name coined by Joel Isaacson in his original patent document[1] describing how fundamental patterns of process arise from the systematic application of operations of distinction and description upon themselves.[2] Louis H. Kauffman has written several background papers on recursion, knotlogic, and biologic.[3]

RD means just what it says. A pattern of distinctions is given in a space based on a graphical structure (such as a line of print, a planar lattice, or a given graph). Each node

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[1] Joel D. Isaacson, "Autonomic String-Manipulation System," US Patent 4,286,330, August 25, 1981, www.isss.org/2001meet/2001paper/4286330.pdf.

[2] See also Joel D. Isaacson, "Steganogramic Representation of the Baryon Octet in Cellular Automata," archived in the 45th ISSS Annual Meeting and Conference: International Society for the System Sciences, Proceedings, 2001, www.isss.org/2001meet/2001paper/stegano.pdf; Joel D. Isaacson, "The Intelligence Nexus in Space Exploration," in *Beyond Earth: The Future of Humans in Space*, ed. Bob Krone (Toronto: Apogee Books, 2006), Chapter 24, thespaceshow.files.wordpress.com/2012/02/beyond_earth-ch24-isaacson.pdf; Joel D. Isaacson, "Nature's Cosmic Intelligence," *Journal of Space Philosophy* 1, no. 1 (Fall 2012): 8-16.

[3] Louis H. Kauffman. "Sign and Space," in *Religious Experience and Scientific Paradigms: Proceedings of the 1982 IASWR Conference* (Stony Brook, NY: Institute of Advanced Study of World Religions, 1985), 118-64; Louis H. Kauffman, "Self-reference and recursive forms," *Journal of Social and Biological Structures* 10 (1987): 53-72; Louis H. Kauffman, "Special Relativity and a Calculus of Distinctions," in Proceedings of the 9th Annual International Meeting of ANPA (Cambridge: APNA West, 1987), 290-311; Louis H. Kauffman, "Knot Automata," in *Proceedings of the 24th International Conference on Multiple Valued Logic – Boston* (Los Alamitos, CA: IEEE Computer Society Press, 1994), 328-33; Louis H. Kauffman, "Eigenform," *Kybernetes* 34, no. 1/2 (2005): 129-50; Louis H. Kauffman, "Reflexivity and Eigenform – The Shape of Process," *Constructivist Foundations* 4, no. 3, (July 2009): 121-37; Louis H. Kauffman, "The Russell Operator," *Constructivist Foundations* 7, no. 2 (March 2012): 112-15; Louis H. Kauffman, "Eigenforms, Discrete Processes and Quantum Processes," *Journal of Physics, Conference Series* 361 (2012): 012034; Marius Buliga and Louis H. Kauffman, "Chemlambda, Universality and Self-Multiplication," in *Artificial Life 14 – Proceedings of the Fourteenth International Conference on the Synthesis and Simulation of Living Systems*, ed. Hiroki Sayama, John Rieffel, Sebastian Risi, René Doursat, and Hod Lipson (Cambridge, MA: MIT Press, 2014); Louis H Kauffman, "Iterants, Fermions, and Majorana Operators," in *Unified Field Mechanics – Natural Science Beyond the Veil of Spacetime*, ed. R. Amoroso, L. H. Kauffman, and P. Rowlands (Singapore: World Scientific, 2015), 1-32; Louis H. Kauffman, "Biologic," *AMS Contemporary Mathematics Series* 304 (2002): 313-40; Louis H. Kauffman, "Self-Reference, Biologic and the Structure of Reproduction," *Progress in Biophysics and Molecular Biology* 119, no. 3 (2015): 382-409; Louis H. Kauffman, "Biologic II," in *Woods Hole Mathematics*, ed. Nils Tongring and R. C. Penner, World Scientific Series on Knots and Everything, Vol. 34 (Singapore: World Scientific, 2004), 94-132; Louis H. Kauffman, "Knot Logic," in *Knots and Applications* (Singapore: World Scientific, 1994), 1-110; Louis H. Kauffman, *Knots and Physics*, 4th ed. (Singapore: World Scientific, 2012).

of the graph is occupied by a letter from some arbitrary alphabet. A specialized alphabet is given that can indicate distinctions about neighbors of a given node. The neighbors of a node are all nodes that are connected to the given node by edges in the graph. The letters in the specialized alphabet (call it SA) are used to describe the states of the letters in the given graph and at each stage in the recursion, letters in the SA are written at all nodes in the graph, describing its previous state. The recursive structure that results from the iteration of descriptions is called RD. Here is an example: we use a line graph and represent it just as a finite row of letters. The alphabet is SA = {=, [,], O} where " = " means that the letters to the left and to the right are equal to the letter in the middle. Thus if we had AAA in the line then the middle A would be replaced by =. The symbol "[" means that the letter to the *left* is different. Thus in ABB the middle letter would be replaced by [. The symbol "]" means that the letter to the right is different. And finally the symbol "O" means that the letters both to the left and to the right are different. SA is a tiny language of elementary letter distinctions. Here is an example of this RD in operation where we use the proverbial three dots to indicate a long string of letters in the same pattern. For example,

…AAAAAAAAABAAAAAAAAAA…

is replaced by

…=========]O[=========…

is replaced by

…========]OOO[========…

is replaced by

…=======]O[=]O[=======….

Note that the element]O[appears and that it has replicated itself in a kind of mitosis. See Figures 1 and 2 for a more detailed example of this evolution. In Figure 3 we show the evolution of the RD, starting from a more arbitrary string. Elementary RD patterns are fundamental and will be found in many structures at all levels. To see a cellular automaton example of this phenomenon of patterns crossing levels of structure, we later look at a replicator in "HighLife" a modification of John Horton Conway's automaton "Life." The HighLife replicator follows the same pattern as our RD replicator. However, the entity in HighLife that is self-replicating requires twelve steps to do the replication. The resultant patterns of replication can be seen in Figures 54 to 61. In the successive figures, twelve steps are hidden and we see the same basic pattern shown in Figure 1. We can understand directly how the RD replicator works. This gives a foundation for understanding how the more complex HighLife replicator behaves in its context. We take this phenomenon of the simple and the complex to be generic for many systems. By finding a point of simplicity, we make possible the evolution of understandings that are otherwise impossible to obtain.

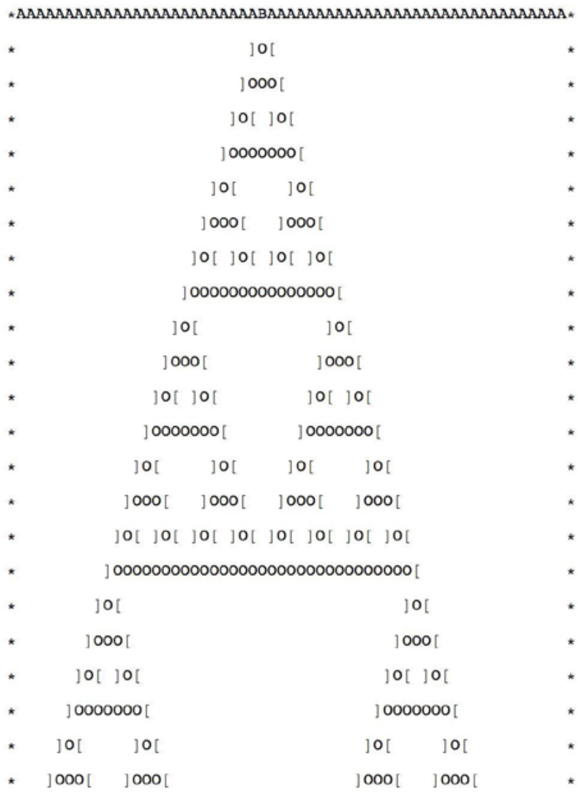

Figure 1: RD replication

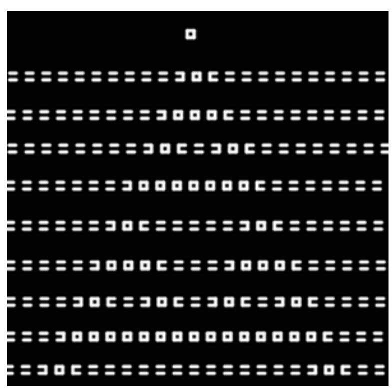

Figure 2: Second picture of RD replication

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*GREEEEEENNONSENSESTREEENG*
*=O[===][]OOOOOOOOOO[=]O=*
*=OO[=]OOO[=========]OOOO=*
*=[]OOO[=]O[======]O[==]=*
*=OO[=]OOOOO[====]OOO[]O=*
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*=O[]O[=]O[=]OOO[====]OO=*
*=OOOOOOOOOOO[=]O[==]O[]=*
*=[=========]OOOOO[]OOOO=*
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Figure 3: A string evolution

We can place the basic idea of RD with the context of cellular automata. RD is distinct from other types of cellular automaton in that its basic recursion is based on direct distinctions made (locally) in relation to distinctions present in the given state of the automaton. In a typical cellular automaton, the next state is obtained on the basis of simple distinctions about the previous state. These distinctions are not necessarily at the letter level. For example, in a Wolfram line automaton we have eight possible local neighborhoods consisting of triples of zeros and ones.

Any distinction made among these eight, separating them into two classes, is acceptable as a rule for the Wolfram automaton. The operation of distinction is shifted to a higher level than the question of sameness or difference for nearby iconic elements of the state. This is the distinction between our “orthodox” RD models and other recursive models. We are interested in rules that involve direct matters of sameness or difference. Such RD rules are very primitive rules. Nevertheless, we regard the orthodox RD models as part of the larger class of recursive cellular automata. We wish to explore the relationships between our primordial structures and the closely related structures of all cellular automata as they are understood at this time.

Everyone who works in science, mathematics, or computer science is familiar with the fundamental role of the concept of distinction and the making of distinctions in both theory and practice. For example, Einstein's relativity depends on a new distinction between space and time relative to an observer and a new unification of space and time that is part and parcel of this distinction. Every moment of using a digital computer depends upon the myriad of distinctions that the computer handles automatically, enabling the production and recording of these words and the computation and transmission of information. Distinctions act on other distinctions. Once a new distinction is born, it becomes the object of further action. Thus grows all the physics that comes from relativity and thus grows all the industry of computation that grows from the idea and implementation of the Turing machine, the programmed computer.

And yet it is not usually recognized that it is through RD that all such progress is made. We discuss RD both in its human and its automatic aspects. In the automatic aspect, we give examples of automata that are based on making very simple distinctions of equality and right/left that then, upon allowing these distinctions to act on themselves, produce periodic and dialectical patterns that suggest what are usually regarded as higher level phenomena. In this way, and with these examples, we can illustrate and speculate on the nature of intelligence, evolution, and many themes of fundamental science.

The remarkable feature of these examples of RD is their great simplicity coupled with the complexity of behaviors that can arise from them. Notice that each successive string in the recursion can be regarded as describing its predecessor. It is remarkable that there should be such intricate structure in the process of description. Description is another word for making a distinction. The description of a given string is a string of individual distinctions that have been made. Each individual distinction is one that recognizes whether a given character in a string is equal to a left neighbor, a right neighbor, both, or neither. This elementary distinction becomes instantiated as a character in the new description string. The description string can be subjected to the same scrutiny, and so the recursive process continues.

Note that this recursive process depends, at its base, on the most elementary distinctions possible for character strings. No mathematical calculations are performed. We should mention that distinction-making without mathematical computation is ubiquitous in natural neuronal processing. Joel Isaacson's collaboration with Eshel Ben-Jacob has included attempts to demonstrate RD in live neuronal tissue.[4] One can also point to the molecular interactions of DNA and RNA as natural RD automata. Finally, we can point to Buliga and Kauffman's[5] notion of chemlambda computation as abstract chemical combination computing that includes aspects of lambda calculus, but is based on direct and local action related to distinctions inherent in the system.

The epistemology behind this automaton is based directly on distinctions that can be made automatic. Other cellular automata are also based on distinctions. For example,

[4] Private Communication with Eshel Ben-Jacob.
[5] Buliga and Kauffman, "Chemlambda."

the well-known Wolfram line automata[6] are based on character strings with only two characters and the recognition of the eight possible triples of characters, including characters to the left and to the right of a given character. The automaton rule then replaces the middle character according to the structure of this neighborhood. There is a crucial difference in epistemology between a Wolfram line automaton and our RD program. We do not replace according to an arbitrary rule. We place a character that describes the distinctive structure of the neighborhood of the predecessor character. Our automaton engages in a meta-dialogue about its own structure. This dialogue is then entered as a string for the automaton to examine and act upon once again. The patterns produced by this recursive distinction are part of a dialogue that the strings hold with themselves. One can ask many questions about RD as presented here. The automaton we have demonstrated illustrates a concept that can be instantiated in many ways. We hope, in a paper to come, to demonstrate Turing universality for automata of this type. But in fact we feel that the paradigm of RD goes beyond (or around) the paradigm of the Turing machine, and we will discuss that issue as well.

There is another level to our automaton and that is the level of examining with human eyes and minds the output of the automaton, seeing patterns in the whole collection of strings and engaging in further design on this basis. This is where the recursive automatic distinctions meet the aware distinguishing of the observers of the system, connecting the automatic with the aware process and design level that goes on in the larger network of science.

It is the case that in the design of computing machines human beings have for centuries confronted the issue of repeatability for the sake of computation or for the sake of the production of pattern (as in weaving) or the reliability of manufacture (as in timekeeping). This means that elementary distinctions must be reproducible and comparable as in mathematical notations, written language, and the mechanics of clocks and computing devices. Thus we shall refer to automatic distinctions when we speak of highly repeatable physical situations that can be regarded as reproductions of distinctions that are available to an observer. In some cases, such distinctions are designed by someone who engineers them into the device. In other cases, we recognize computational and reproducible patterns in natural situations. The earth goes around the sun periodically; the moon goes around the earth. Natural clocks arise from these periodicities and regularities observed in our world. Thus, in this essay, we do not restrict ourselves in the use of the word distinction to the meaning that a distinction is made by some human observer. We refer to distinctions that are ongoing in a device beyond our direct observation. Nevertheless, the buck stops at a human observer who recognizes the patterns of the device and who interprets the meaning of what has been produced. It is then possible to discuss the role of creativity in relation to deterministic and automatic actions.[7]

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[6] Stephen Wolfram, *A New Kind of Science* (Champaign, IL: Wolfram Media, 2012).

[7] Acknowledgement. We thank Bernd Schmeikal for conversations and for sharing his own research in relation to our work. We thank Dan Sandin for a continuing collaboration with Lou Kauffman and particularly for sharing the computer program for 2D RD that has been evolved by the two of them. The graphical illustrations of 2D RD in this paper were all produced by that program. It also gives us great

2. The Logic of Distinction and the Distinction of Logic

We have introduced the one-dimensional RD and its very simple alphabet based on the four iconic symbols shown in Figure 4. In this figure, we use a box rather than a circle for the icon that indicates difference to both the right and the left, and we use a box with a missing left vertical edge to denote sameness on the left and a box with a missing right vertical edge to denote sameness on the right. Sameness on both right and left is indicated by the two parallel lines that remain when the two vertical edges of the box are removed. In this figure, we give a logical justification for these icons in terms of the act of discrimination. That is, we give a logical construction for an icon that describes and embodies the discrimination itself. At a given point in the line of letters, there is a given letter. This letter is either distinct or different from its neighbor to the left and/or its neighbor to the right. We introduce a method to *manufacture* an icon that expresses these distinctions. In order to do this, we insert a line segment in between the space for the given letter and the space next to it *if there is a difference between the given letter and its neighbor*. We take as given a line segment at the top of the space and a line segment at the bottom of the space. (This actually indicates the condition of the one-dimensional RD where it is distinct from its context above and below the one dimension of operations.) As a result, this process of discrimination constructs four possible icons that describe the condition of a given letter. The icons are illustrated (Figure 4), and the reader can see that they are an equals sign when there is no distinction to the left or to the right, left and right brackets when there is a distinction to the left or the right but not both, and a rectangular box when there is a distinction to both the right and the left. In the next few paragraphs, we describe this process further in terms of logical operations.

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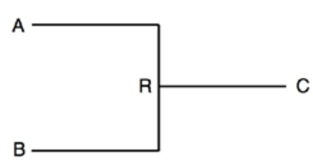
pleasure to acknowledge Tom Mandel. Fifteen years ago, on his own initiative, he posted Joel Isaacson's patent and his Stegano paper on the ISSS website, when he managed that site. Furthermore, we feel that the basic RD process is a clapping machine realizing part of Tom's vision for the notion of depicting a relationship as a picture where when the This and the That are the two hands, then the Clapping of the hands connotes the relationship that is brought forth. In the RD, it is the distinctions and the spaces between them that clap in time and produce the "sounds" of further distinctions. Tom uses the notation shown below,



or algebraically, *(A,B)R = C,* where C stands for the (whole) dividing/arising from A and B, and R the connection/relation of A and B; the This and the That. Such notation is simple, yet insistent, calling for the articulation of the unity C, the relation R and the "parts" A and B: Why is this important? The answer is: Because such notation and the attitude behind it continually call the question of relationship and the nature of relationship. All descriptions, all systems, are built this way. But we keep forgetting the glue and putting it into the background. Here, all three fundamentals in any distinction are brought into the foreground.

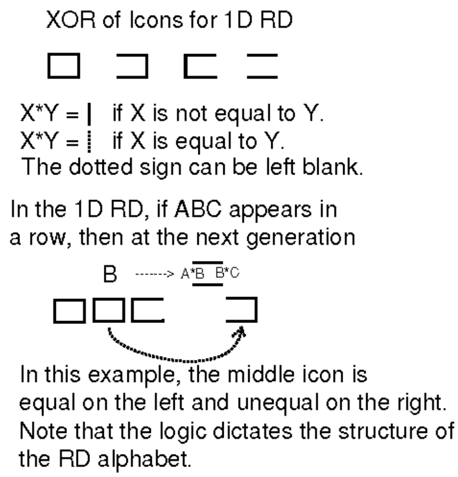


Figure 4: XOR of icons

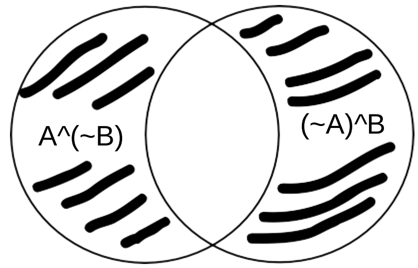
The fundamental underlying operation is exclusive or, often denoted by XOR. When we say “A XOR B” we mean the statement “A or B but not both A and B.” This special version of OR has the property that it is true only when A and B have different truth values. Logically, “A or B but not both A and B” is equivalent to “A and not B, or B and not A.” In this form we write the formula

$$A * B = (A \wedge (\sim B)) \vee ((\sim A) \wedge B).$$

Here $A * B$ denotes “A XOR B”, $A \vee B$ denotes “A or B,” and $A \wedge B$ denotes “A and B.”

When working with sets, we can interpret $A * B$ as the intersection of A with the complement of B taken in union with the intersection of the complement of A with B. This is illustrated in Figure 5. In using the Venn diagrams, we have a very intuitive interpretation of XOR. A set is denoted by a shaded circle and when we XOR two sets, the part where they overlap vanishes. Thus two identical sets will yield an empty diagram under this operation. In this sense, a set is its own negation! We return to this point of view in Section 10 when we discuss the relationship of RD with Spencer-Brown’s Laws of Form. In letting one shaded region operate upon another, the parts that remain black after the XOR operation indicate the differences between the two sets. In this way, XOR is a logical exemplar of the operation of discrimination and it can be understood to underlie all the RD operations we describe. One can imagine that discrimination (as practiced by thinking beings) is more complex than XOR, but XOR is a backbone or skeletal aspect of all instances of discrimination.

A or B but not both A and B.



A XOR B = A*B = A^(~B) v (~A)^B

Figure 5: XOR in Venn diagrams

Now view Figure 4 once more. Here we show explicitly how the XOR operation acts on the icons for the 1D RD to produce the icons at the next iteration. We use a vertical slash **|** and an unmarked vertical slash for the two states of discrimination. We call these the *marked* and *unmarked* states, respectively. Given two such states, we define A * B as marked if one of A and B are different. If A equals B, then A * B is unmarked. This construction is then applied to the local interactions of the icons in the RD. If we have a row with ABC in that row, then for the new B, we form A * B and B * C. These vertical slashes or unmarked slashes become the left and right ends of the new icon that represents the new B in the next row of the RD, one full time-step later. Thus the new icon is formed by the discriminations to its left and to its right in regard to those neighbor icons. The figure shows explicitly how we leave the horizontal lines of the icon unchanged while we change the vertical slashes. As mentioned at the beginning of this section, this means that the logic of left and right naturally creates the four icons that are used in the 1D RD. The alphabet arises in the act of discrimination. The act of discrimination is quite general for the RD. Any letters or icons can be given to it at the start. The XOR applies to make the discrimination and to produce a standard icon that indicates the left-right discrimination that was made.

Now view Figure 6, where we indicate how the XOR process can be accomplished by digital circuitry. The figure should be self-explanatory. There is a basic inverting element that will take states to their opposites and, with a multiplicity of inputs, this inversion is regarded as a NOR gate. That is, one starts with a collection of variables {a, b, c, d} and the NOR gate returns $\sim (a \vee b \vee c \vee d)$. The circuit then implements the formula for the XOR operation that we have given above. This means that we could have an RD automaton that sampled signals inside a larger digital environment. It also means that we can look at the RD as connected inside an information-processing environment that uses logical operations in great generality. In particular, one could think of a sensing device that can detect differences in signals with which it otherwise has no direct access. Isaacson [8] has called such external but not directly detectable signals *fantomarks*. The information about their differences can become the initial data for an RD system that then amplifies and modifies these patterns, allowing the possibility for communication (by letting another system find differences in the signals generated by

[8] Isaacson, “Autonomic String-Manipulation System.”

this RD) between systems that have internal states that are fantomarked for the other system. Isaacson has speculated that this could be the basis for communicating with extraterrestrials. Here we point out that it can be regarded as a partial description of the situation of human-to-human communication with its mix of local-to-global discrimination based on the detection and articulation of differences.

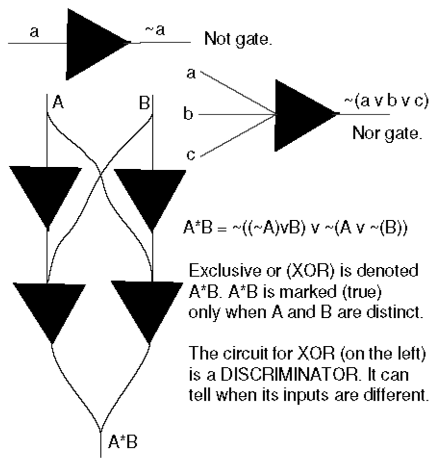


Figure 6: XOR circuit

We regard this description of the process of discrimination to be fundamental. A ground that is subject to discrimination is given at the beginning. The XOR operations probe this ground and write naturally via marked and unmarked states in the geometry and alphabet of special icons that can be further discriminated by the same process. The icons record a neighborhood of discriminations. In the case of 1D RD, this neighborhood is described in terms of left and right. The process of discrimination alternates between the local indications of marked and unmarked states (the vertical slash and its absence) and the global examination of icons for their identity or difference. It is this crossing of levels that makes the structure of the RD process repeatable and unique.

In general, an RD structure has alphabetic elements at specific loci. A process of discrimination generates an icon for that location that describes the distinctions between that letter and its neighbors. These icons of distinction become the letters of a special alphabet that is coherent with the geometry of the RD structure. The recursion replacing present icons or alphabetic elements with these icons of distinction is the process of RD. The process arises directly from the idea of description and the fundamental distinction of the given geometry. In the next section, we show how this works for two-dimensional RD.

3. Two-Dimensional RD, a 16-Letter Alphabet, Quaternions and Spacetime

We now consider a natural generalization of the one-dimensional RD to two dimensions. The geometry of the 2D RD is a rectangular lattice with square cells. Each cell is regarded as having four neighbors, one to the north, one to the south, one to the east, and one to the west, each sharing a one-dimensional interval of common boundary. The simplest occupant of such a cell corresponds to openings or closings of the four parts of the boundary. Thus one can block all of the boundary, or all but one edge of the boundary, or all but two edges of the boundary and continue until one has the unique empty icon with no edges from the boundary. This makes a 16-letter alphabet, as illustrated in Figures 7 and 8.

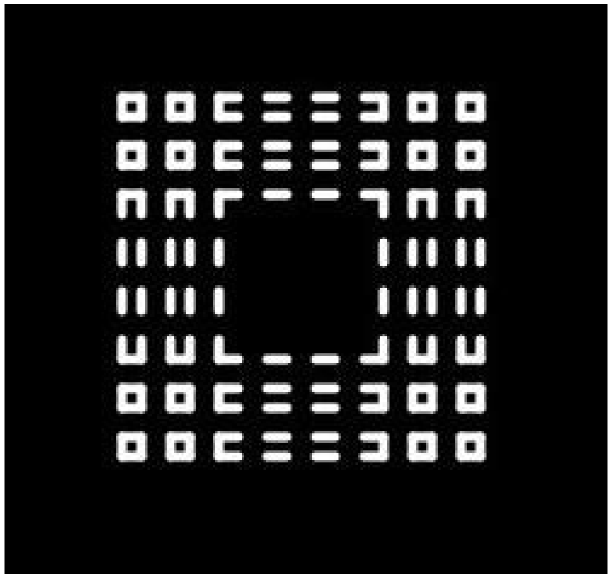


Figure 7: A snapshot of a 2D RD

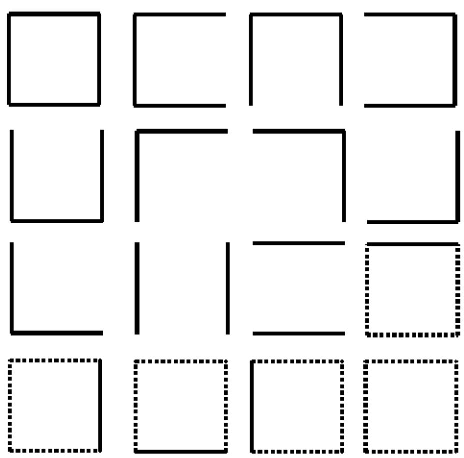


Figure 8: The 2D alphabet 1

In Figures 9 and 10, we indicate how to code the letters as ordered sequences of four elements, each element a plus or a minus sign. In these figures, we also indicate how to make XOR combinations of these edges of the icons. The rule is simply that the superposition of two edges cancels them. With this, we can combine the letters to form other letters by superimposing them. When two letters are identical, then the superposition is the empty letter. Otherwise it is not empty, and it is a new resultant letter. Thus, we see that this superposition of letters serves to distinguish one letter from another. Two letters are distinct if and only if their superposition is empty.

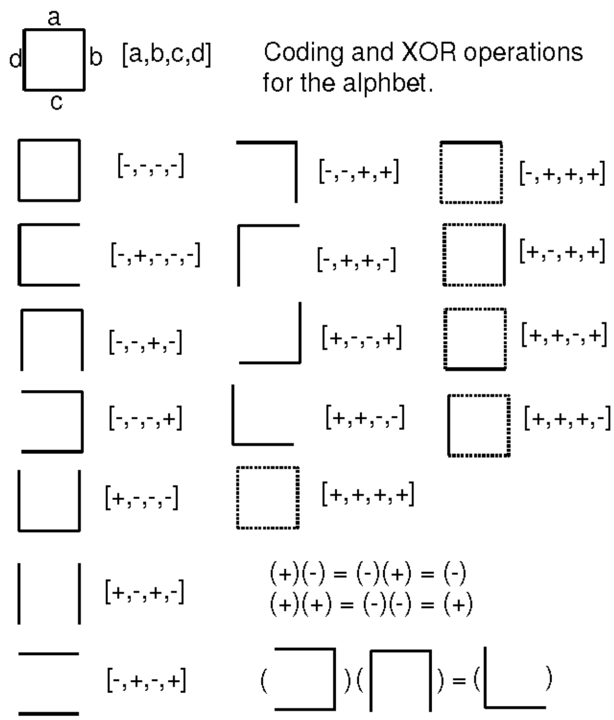


Figure 9: The 2D alphabet 2

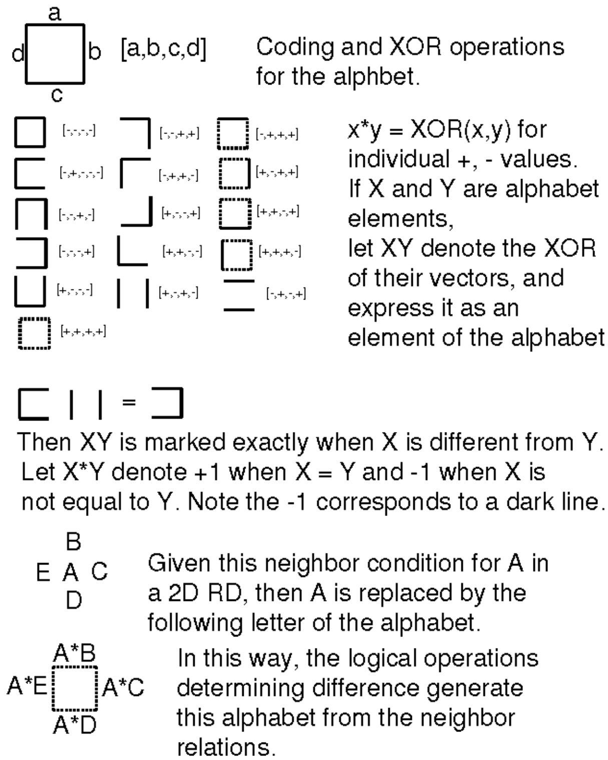


Figure 10: The 2D alphabet 3

In the sequence from Figure 11 to Figure 19, we show eight steps from the first figure and returning to that figure. The first figure is an empty box with a fixed boundary condition that declares that its outer squares are different from the adjacent squares outside the box. Each successive figure is the result of one redescription by the RD process. In this case and with this initial condition, the process has period eight.

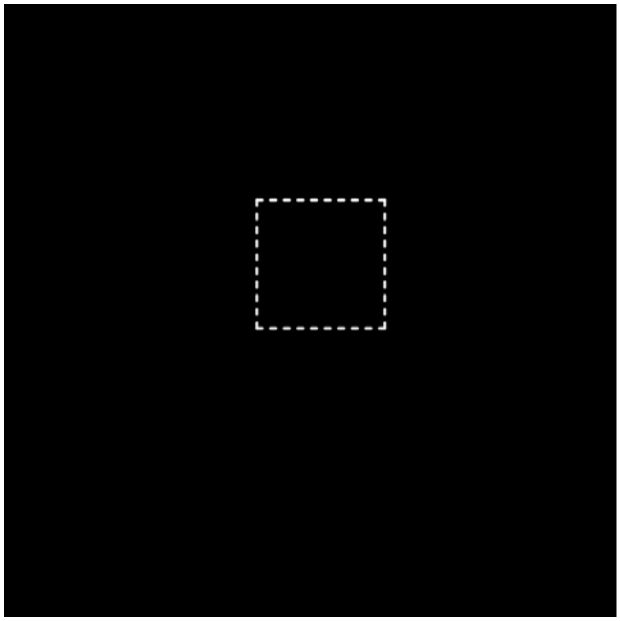


Figure 11: 2D RD box, no seed

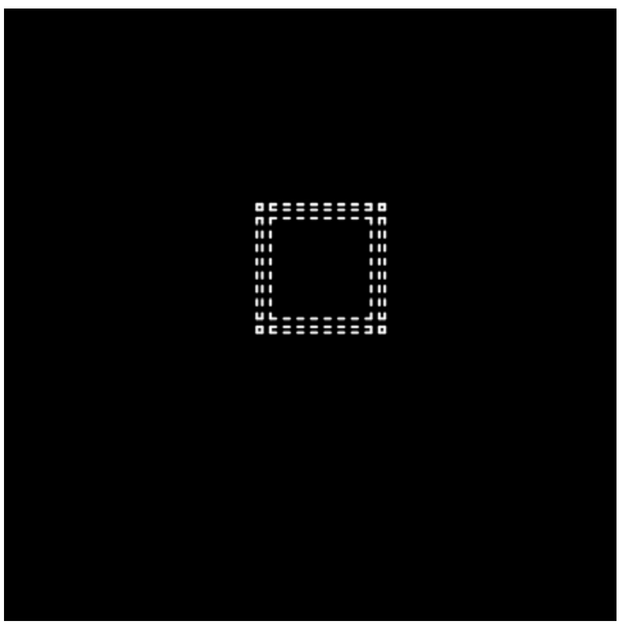


Figure 12: 2D RD box, no seed

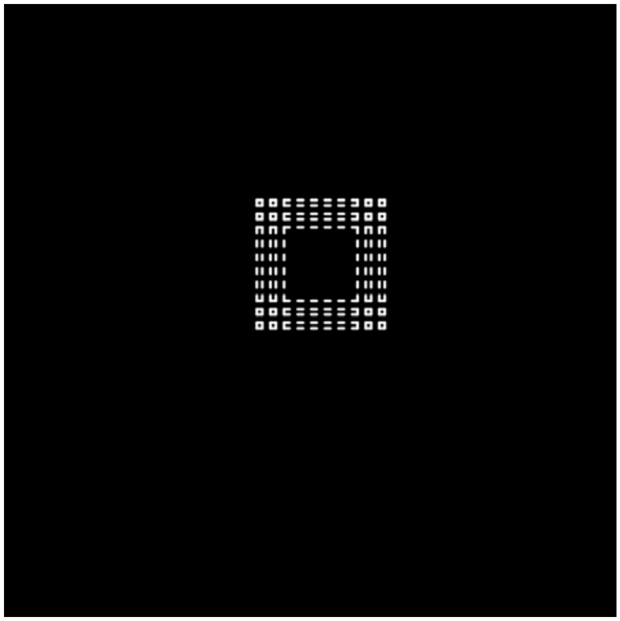


Figure 13: 2D RD box, no seed

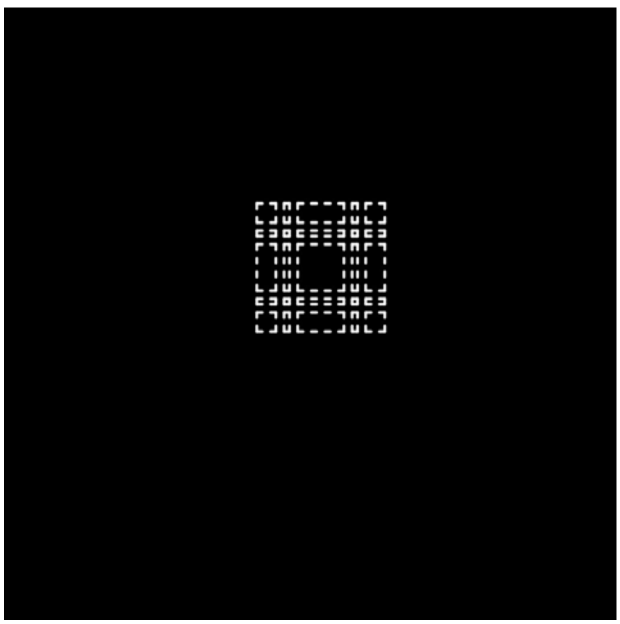


Figure 14: 2D RD box, no seed

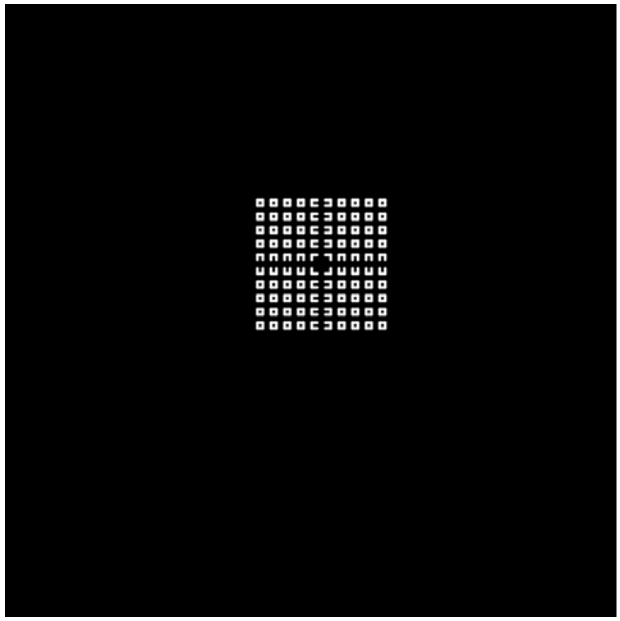


Figure 15: 2D RD box, no seed

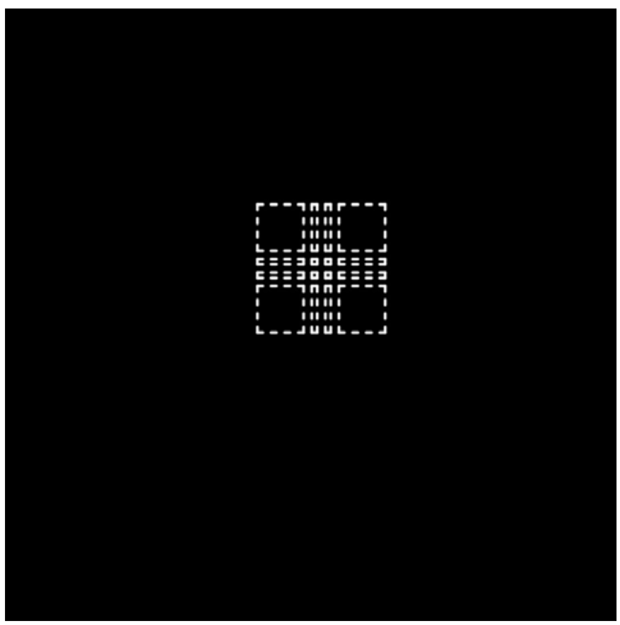


Figure 16: 2D RD box, no seed

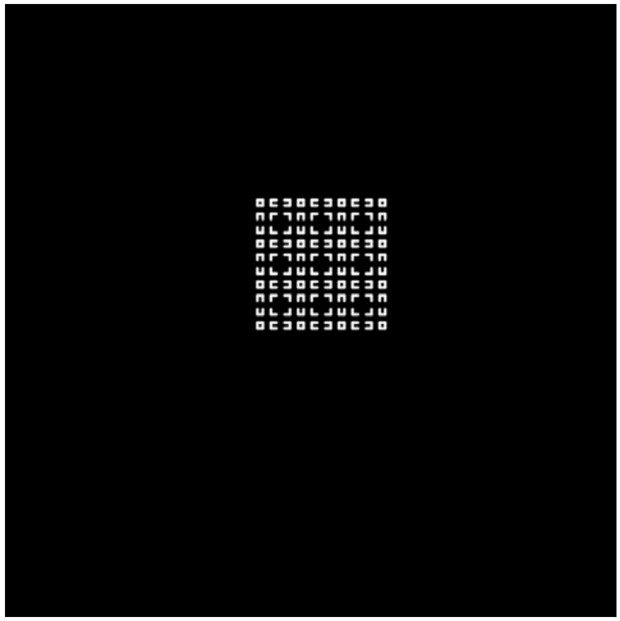


Figure 17: 2D RD box, no seed

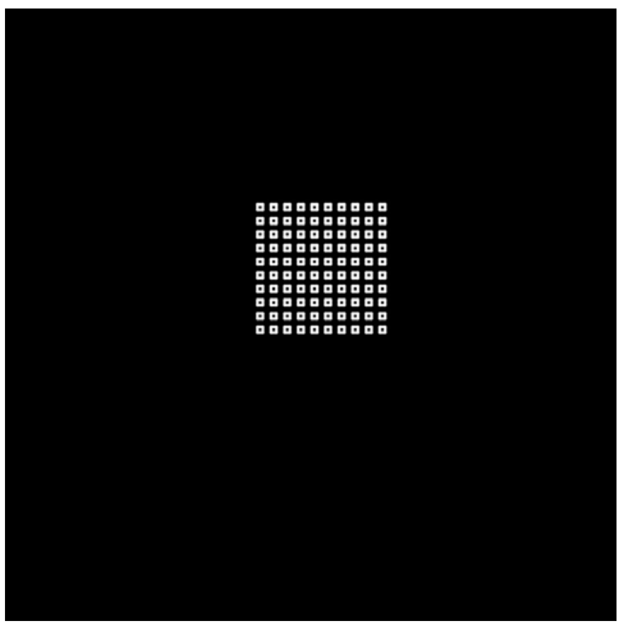


Figure 18: 2D RD box, no seed

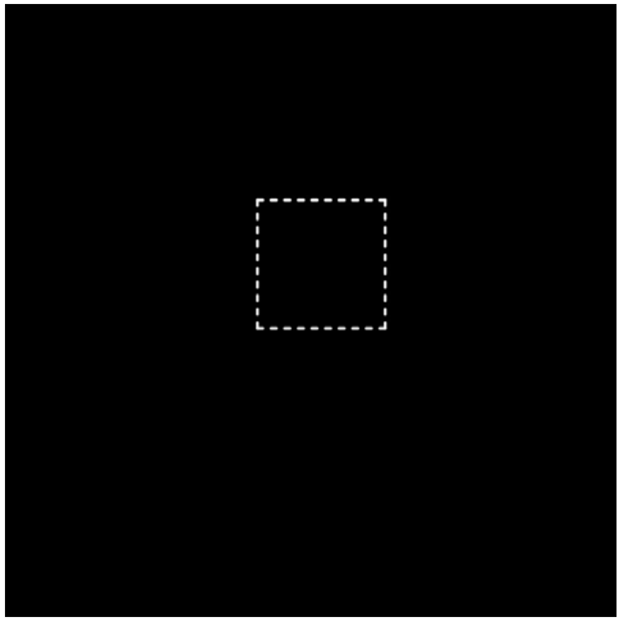


Figure 19: 2D RD box, no seed

In the sequence from Figure 20 to Figure 32, we show the same box with a different initial condition (some marked spaces inside). Now the evolution is more complex, as is illustrated in the figures. Remarkably, in this case the result is eventually periodic of period two.

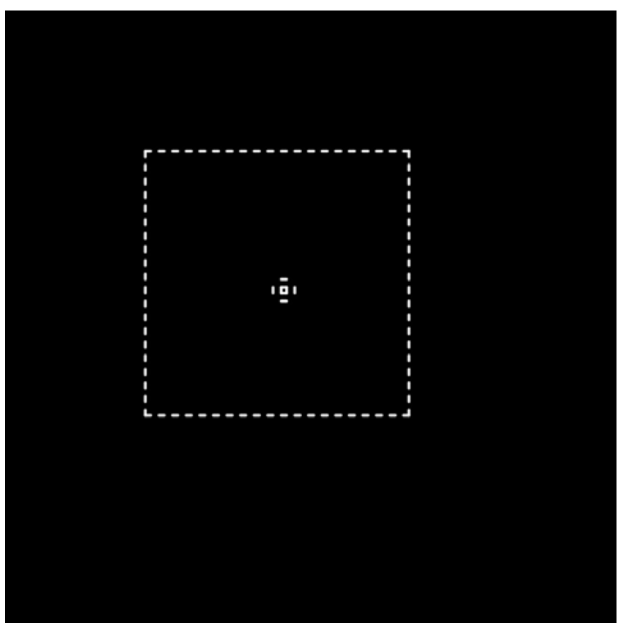


Figure 20: 2D RD box with seed

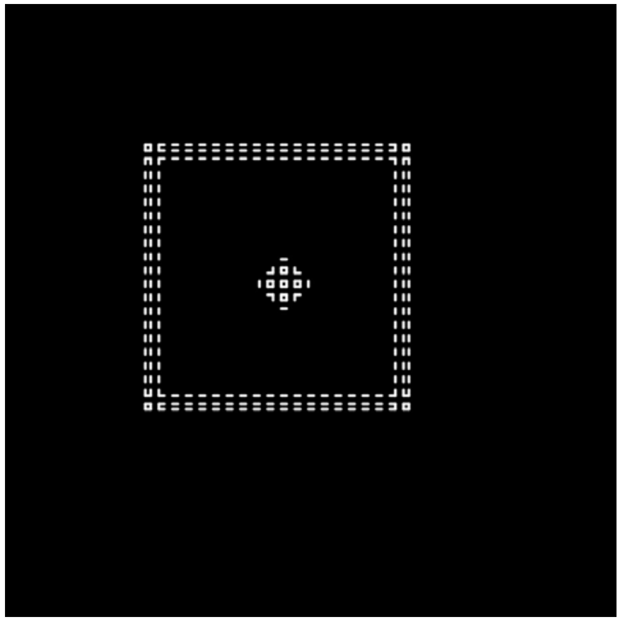


Figure 21: 2D RD box with seed

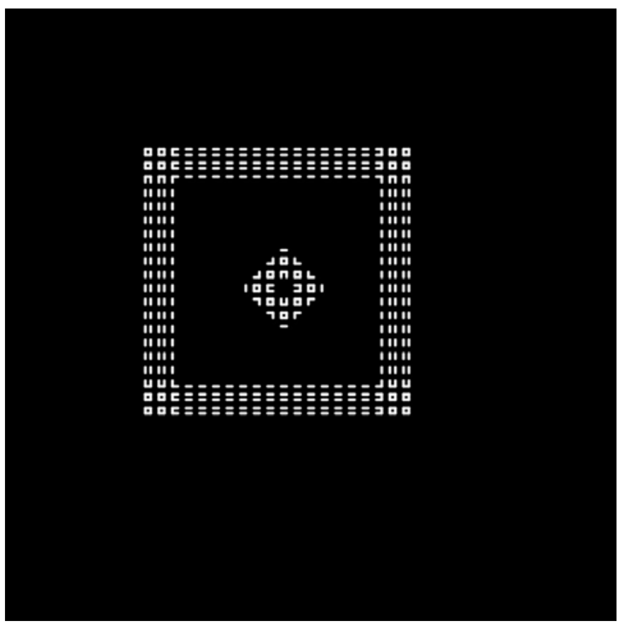


Figure 22: 2D RD box with seed

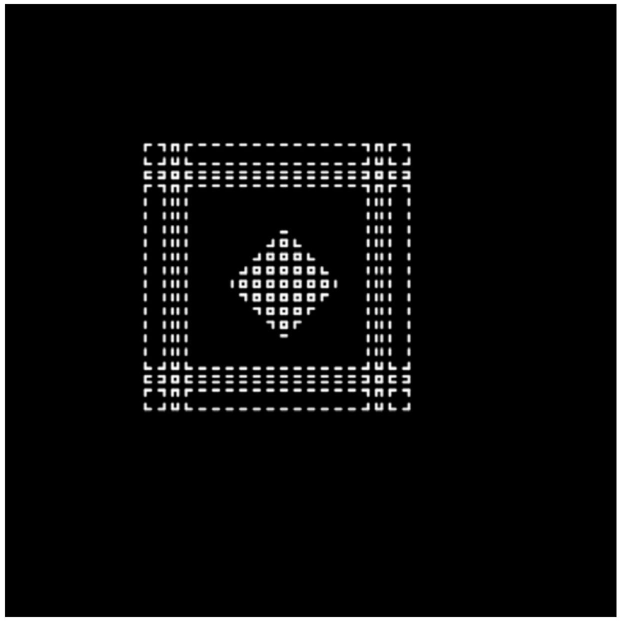


Figure 23: 2D RD box with seed

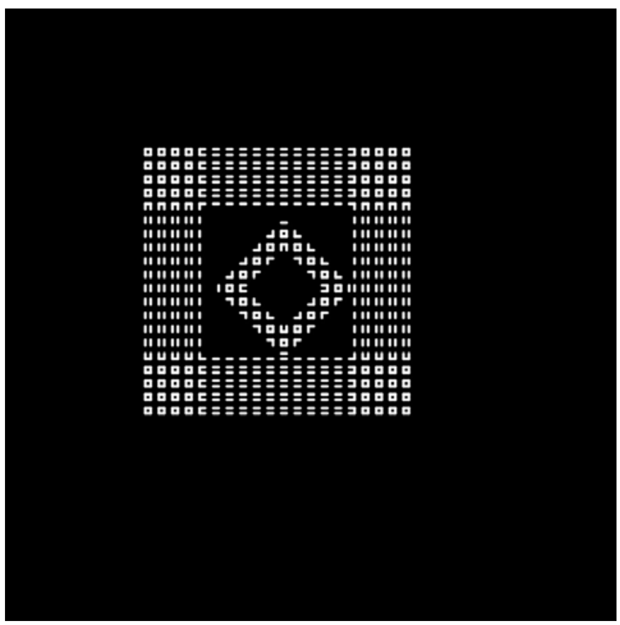


Figure 24: 2D RD box with seed

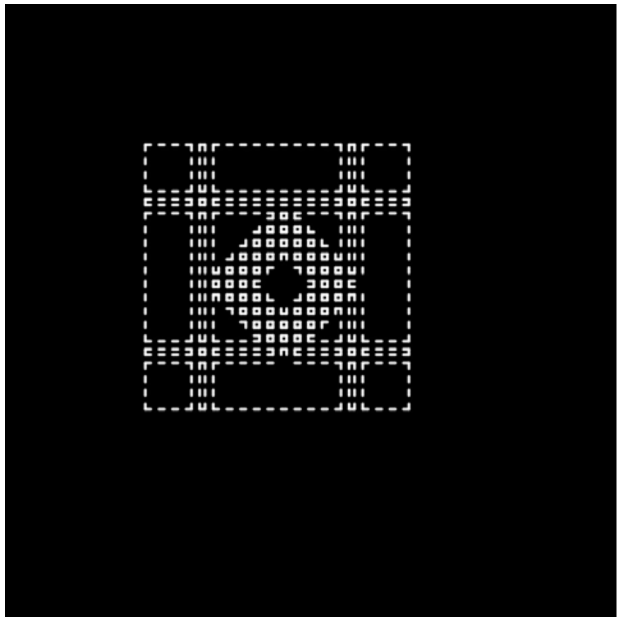


Figure 25: 2D RD box with seed

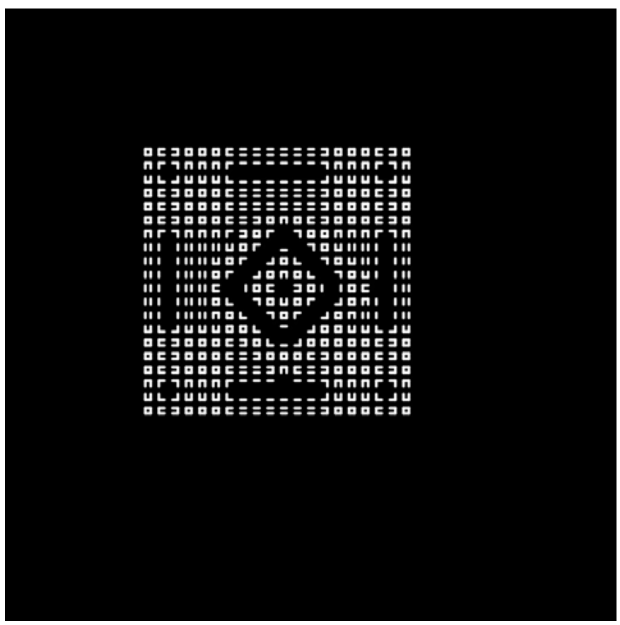


Figure 26: 2D RD box with seed

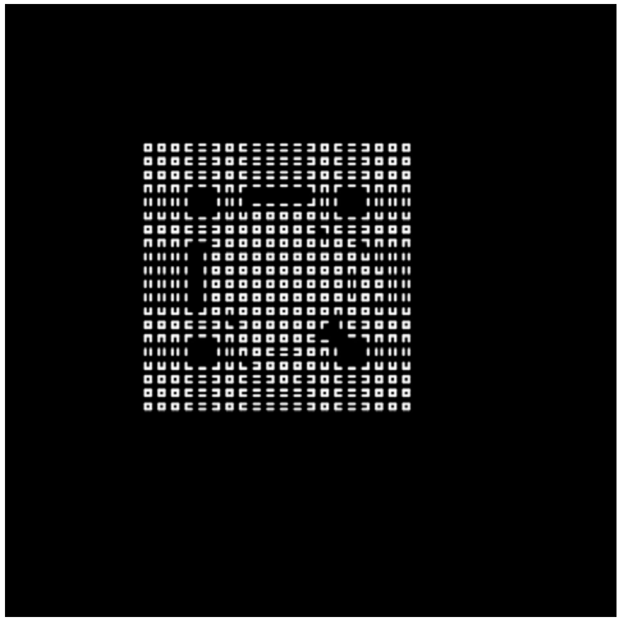


Figure 27: 2D RD box with seed

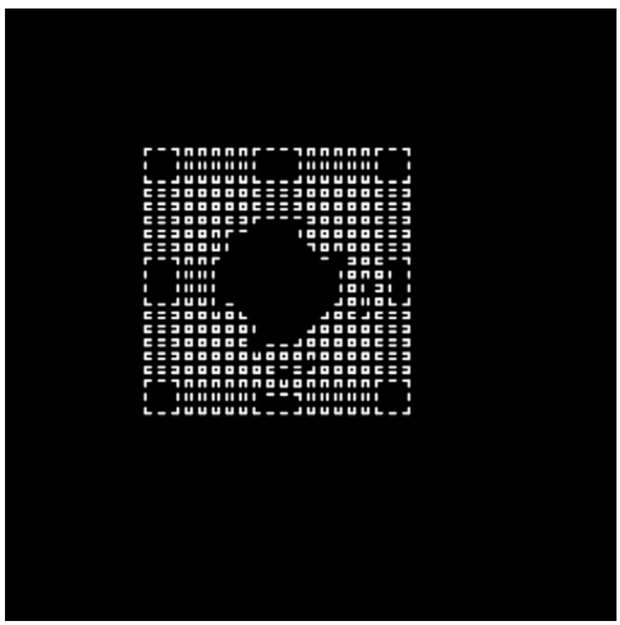


Figure 28: 2D RD box with seed

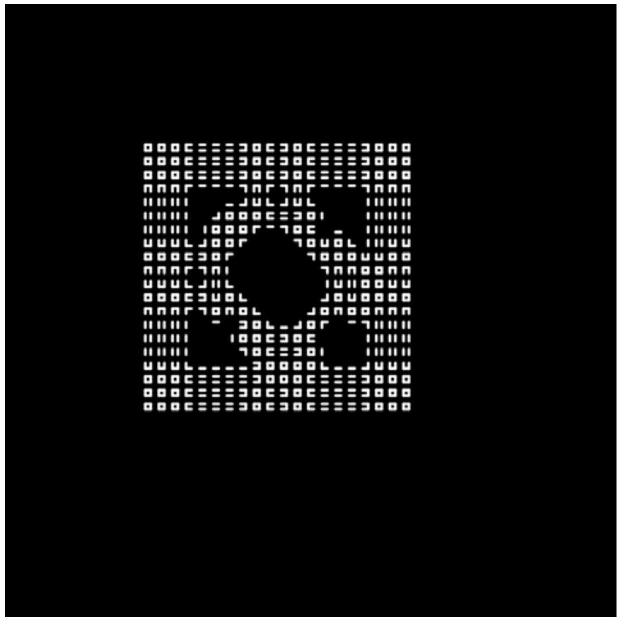


Figure 29: 2D RD box with seed

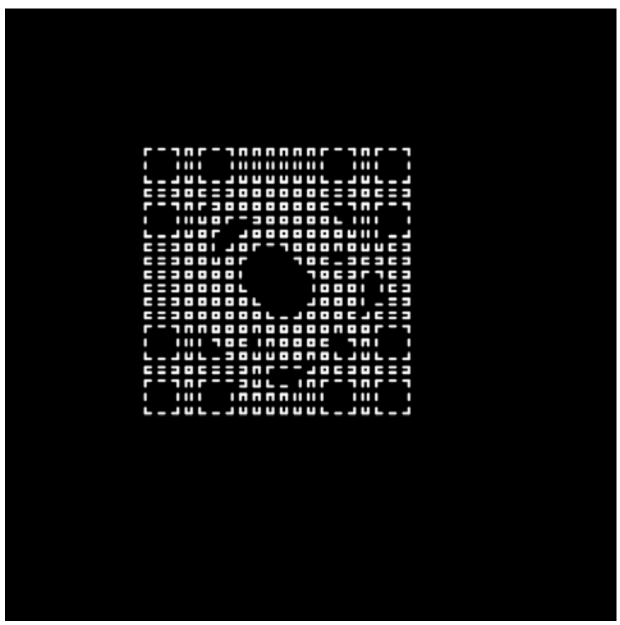


Figure 30: 2D RD box with seed

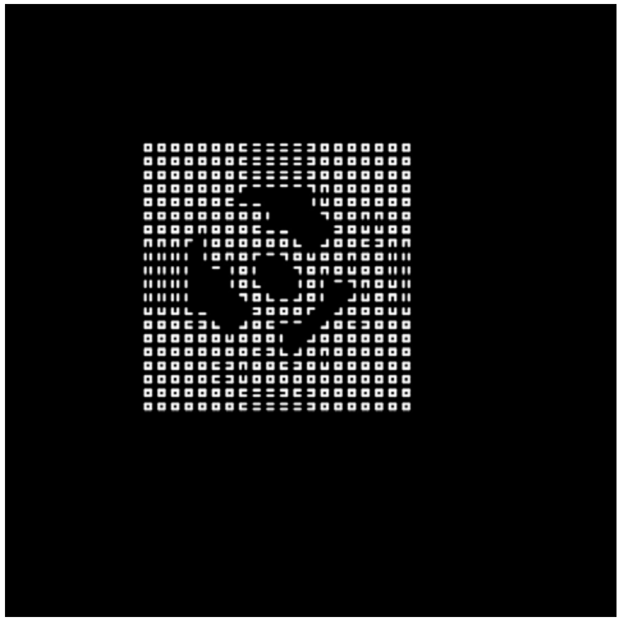


Figure 31: 2D RD box with seed

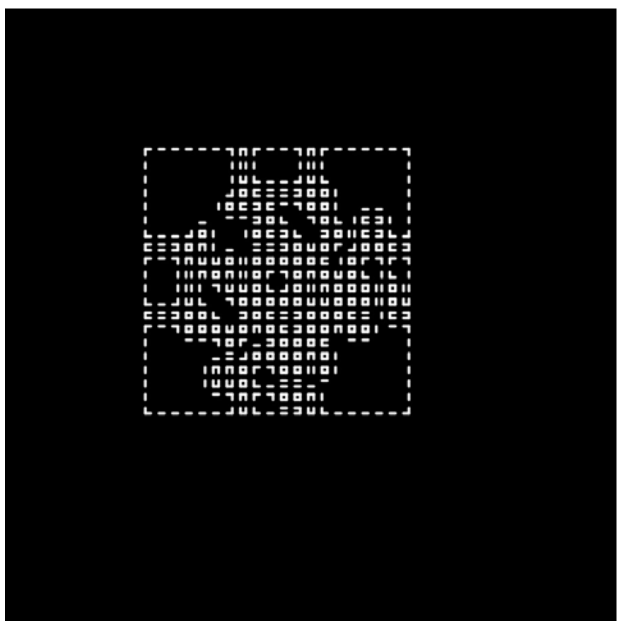


Figure 32: 2D RD box with seed

Figure 33 and Figure 34 illustrate two consecutive frames from this automaton after it has entered period two. The reader can compare these two frames and see that each describes the other. Focus on the pair of 2D patterns, Tweedledum and Tweedledee, in these two figures. What is remarkable about these two patterns is that they mutually describe each other in such a way that they complement each other, just like a positive and a negative in photography. If separated, each would construct its complement, and the patterns would replicate indefinitely. So these are antithetical and their superposition yields a synthesis. (A synthesis here would be the big square filled completely with only little squares.) Note that they are typical in many 2D RD runs and are not exceptions.

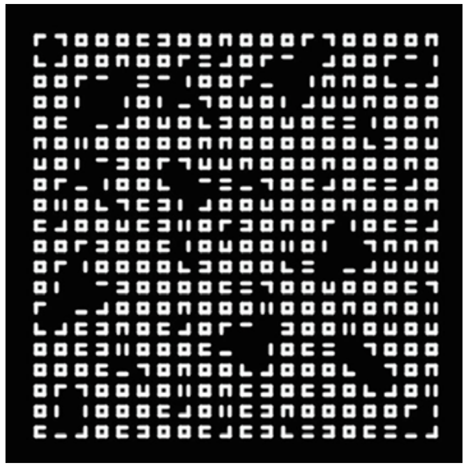


Figure 33: Tweedledum

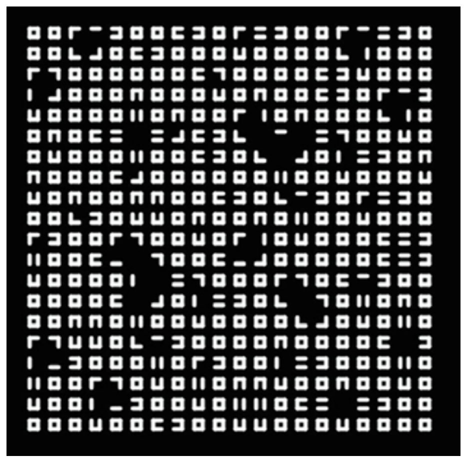


Figure 34: Tweedledee

The two strands of DNA are also complementary, which allows their replication. The reader will recognize how much more complex this 2D complementarity is than the 1D complementarity of DNA. Obviously, no one can dream of or design such intricate mutual descriptions of patterns, and yet they are by-products of an automatic RD automaton. One might speculate that the DNA molecule with its complementary Watson and Crick strands evolved through recursive chemical interactions.

3.1 *Quaternions and Iterants*

In this subsection, we show how the 16-letter alphabet is related to the algebra of the quaternions and concomitantly to the algebra of spacetime. Before we do this, however, it will be helpful to explain a way to think about such matters that is developed in the paper by Kauffman (and the references therein).[9] In that paper, one finds a temporal interpretation of the square root of minus one. The idea is that one starts with a simple oscillation such as

$$\ldots+-+-+-+-+-\ldots.$$

[9] Kauffman, "Iterants, Fermions, and Majorana Operators."

Starting in this way, we can connect with RD simply by observing that some of the simplest 1D RD with tight boundary conditions will oscillate with period two. Once recursion is on the scene, the simplest oscillations are inevitably present. That said, let us make two abbreviations that correspond to two ways to distinguish a period two oscillation:

$$[+, -] = [+1, -1]$$

and

$$[-, +] = [-1, +1].$$

These two ordered pairs correspond to distinguishing the oscillation as proceeding from plus to minus or as proceeding from minus to plus.

Call an ordered pair such as [a, b] an *iterant*. We can combine iterants by adding their coordinates or by multiplying their coordinates.

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b][c, d] = [ac, bd].$$

We add to this structure an operator η that participates in the *time shift* that relates one iterant to the other.

$$\eta^2 = 1$$

$$[a, b]\eta = \eta[b, a].$$

Formally, η acts as a permutation of order two, exchanging [a, b] for [b, a] when it is commuted with an iterant. We regard an element of the form [a, b]η as a *temporally sensitive iterant*. Note what happens when we multiply

$$i = [+1, -1]\eta$$

by itself.

$$i^2 = ii = [+1, -1]\eta[+1, -1]\eta = [+1, -1][-1, +1]\eta\eta = [(+1)(-1), (-1)(+1)]1 = [-1, -1] = -1.$$

Thus

$$i^2 = -1.$$

We have produced a square root of minus one as a temporally sensitive iterant associated with an elementary oscillation.

In fact, we have produced an algebra containing

$$\{\eta, 1 = [1, 1], -1 = [-1, -1], \alpha = [1, -1], -\alpha = [-1, 1]\}.$$

Note that

$$\eta^2 = \alpha^2 = 1$$

and that

$$\alpha\eta + \eta\alpha = 0.$$

This is a first example of a *Clifford algebra*, an algebra generated by elements of square one that anti-commute with one another. We have $i = \alpha\eta$ and

$$i^2 = \alpha\eta\alpha\eta = \alpha(-\alpha)\eta^2 = -\alpha^2 = -1.$$

Thus, we can also see our temporal interpretation of the square root of minus one as a Clifford algebra phenomenon.

Clifford algebras are deeply connected with physics. To see a hint of this we consider a fundamental formula from special relativity theory (we use the convention that the speed of light is $c = 1$.). Let E denote energy, p momentum, and m the mass of a particle. Now let

$$E = \alpha p + \eta m.$$

Assume that p and m commute with α and β. You can easily prove by multiplying it out that

$$E^2 = (\alpha p + \eta m)(\alpha p + \eta m) = \alpha^2 p^2 + \eta^2 m^2 + (\alpha\eta + \eta\alpha)pm = p^2 + m^2 + 0pm = p^2 + m^2.$$

This formula $E^2 = p^2 + m^2$ is fundamental to special relativity, and we have shown that it follows from a Clifford algebra representation of the energy. This way of writing the energy is due to the great physicist Dirac, and is the beginning of the deep relationship between Clifford algebra and physics. Our point is that by looking at this through the lens of iterants, we can draw a connection between fundamental recursion and quantum and relativistic physics.[10]

Now we turn to the *quaternions.* Sir William Rowan Hamilton discovered quaternion algebra in 1843, after 15 years of trying to find a three-dimensional analog for complex numbers. When he realized the key was a four-dimensional space, the pattern fell into place. Recall that the quaternions are generated by $\{1, -1, I, J, K\}$ so that $I^2 = J^2 = K^2 = IJK = -1$ from which it follows that $IJ = K$, $JK = I$, and $KI = J$, and that $IJ = -JI$, $JK = -KJ$, and $KI = -IK$.

There is a natural iterant structure for the quaternions (see Figure 35). In this figure we show the order four iterant sequences that correspond to each of I, J, and K and the analogy of the simple time shifter η that is associated with each one. These analogs are

[10] For further details, see Bernd Schmeikal, "Basic Intelligence Processing Space," *Journal of Space Philosophy* 5, no. 1 (Spring 2016): 65-89; Kauffman, "Iterants, Fermions, and Majorana Operators."

diagrammed as permutations, and they act when one composes the iterants by attaching their braided forms together. The new temporal shift operators generate the so-called Klein Four Group, the symmetries of a square.[11] We now show how this iterant version of the quaternions is related to our 16-letter alphabet and how the symmetries of the square come into play directly.

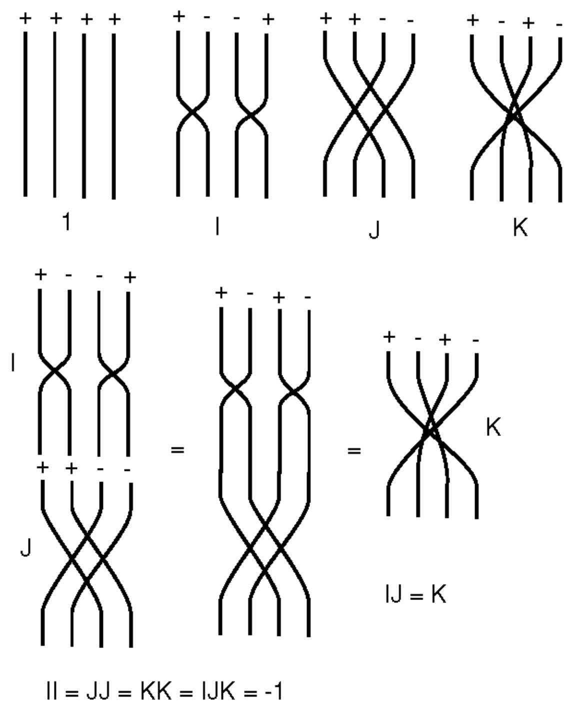


Figure 35: Iterant representation of the quaternions

Now we turn to Figure 36, where we show how there is a natural quaternion structure associated with the 16-letter alphabet. What you see is a subset of the 16-letter alphabet and the operations A, B, C (and 1) of the Klein Four Group. We define I, J, K each of the form I = aA, J = bB, and K = cC where a, b, and c are certain elements of the 16-letter alphabet. We then define, e.g., $xA = Ax^A$, where x^A is the operation of the symmetry element A on the letter x. We define xy (on letters) via XOR of the corresponding letters in the alphabet. We find that I, J, and K give the quaternions. Thus the quaternions are a combination of XOR operations and symmetry operations in the alphabet. Note that xy = XOR(x,y) = the result of superimposing x and y as letters and canceling common occurrences. Once we have the quaternions, we have an entry into spacetime algebra as follows. We have II = JJ = KK = IJK = −1. Let E = (x,y,z,t) = xI + yJ + zK + t1 where x, y, z, and t are real numbers. Then think of E as a point in spacetime. We have

$$E^2 == (xI + yJ + zK + t1)(xI + yJ + zK + t1) = -x^2 - y^2 - z^2 + t^2$$

[11] See Kauffman, “Iterants, Fermions, and Majorana Operators” for more details.

Which is the Minkowski metric (it is often written as the negative of this expression) for spacetime.

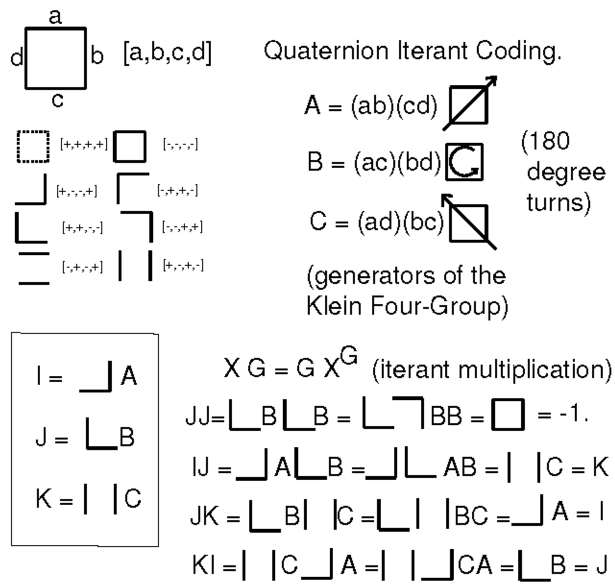


Figure 36: The 2D alphabet 4

Electromagnetism and much other physics can be written in quaternionic language. One can start with a Clifford algebra with generators e_1, e_2, e_3, e_4 with $(e_i)^2 = 1$ and distinct elements anti-commuting and construct spacetime algebra, the quaternions, and more. The iterant structure that we have hinted at here is part of a reformulation of the mathematics of matrix algebra that puts it into a temporal framework and a framework that respects the ubiquitous appearance of the symmetries of permutation groups. It is likely that in another generation of the RD concept, we shall include more about the role of symmetry. In this way, we have the beginnings of a relationship of RD structure and fundamental frameworks for physical theory.[12] All this said, we have made only a superficial connection between the spacetime algebra of the quaternions and the actions or operations of the 2D RD.

The iterant process is in back of the quaternion multiplication, where the symmetry group acts on the alphabetical letters. This could become part of an extension of RD operations. Then the RD would not just compare and describe. It would also interact with its own descriptions and change them by certain symmetry operations. This is one possibility for adding rules, but we do not yet have a clear picture of what extra structure can be added naturally to the very simple base with which we have started.

[12] For further details, see Schmeikal, "Basic Intelligence Processing Space"; Kauffman, "Iterants, Fermions, and Majorana Operators."

4. Distinctions, Distinctioning and Wolfram Automata

In this section, we make a comparison with the general structure of Wolfram line automata.[13] The Wolfram automata use a very simple alphabet consisting of two letters (black and white, or 0 and 1). At every stage in the process, a distinction is applied to the eight possible states consisting of a square and its neighbors to the left and to the right. The distinction assigns 0 or 1 to each of these states, and the fate of the middle square in the next row is decided by that distinction. We see that these line automata are certainly RD automata, but that they are not strictly orthodox in our sense, in that the alphabet is not descriptive of all the local distinctions under consideration. The alphabet is simple, but the distinctions that can be made are complex. The result of this choice leads to a large and interesting body of phenomena.

In Figure 37, we see a depiction of the results of applying Wolfram Rule 126. As the reader can see, by comparison with Figure 1 and Figure 2, the overall pattern resulting from Rule 126 is essentially the same as that obtained from our 1D RD. The underlying structure of alphabet and distinction is different. This is a first example indicating the need for more detailed comparison between orthodox RD rules and cellular automata. We will leave such analysis for further work. In Figures 38 and 39 we illustrate Rule 110 and show how its iteration looks. It differs from Rule 126 in only one place. This de-symmetrization of Rule 126 results in very complex behavior. Here we are farther from the simple 1D RD. Rule 110 takes full advantage of the very simple alphabet of zero and one, and it uses an asymmetrical distinction on the set of eight triples of zeros and ones. The result is a very complex pattern of evolution and an automaton that has been proved to be Turing universal. One can certainly regard Rule 110 as a highly successful application of non-orthodox RD. We will return to this rule in a subsequent paper and examine it further in the light of RD structure.

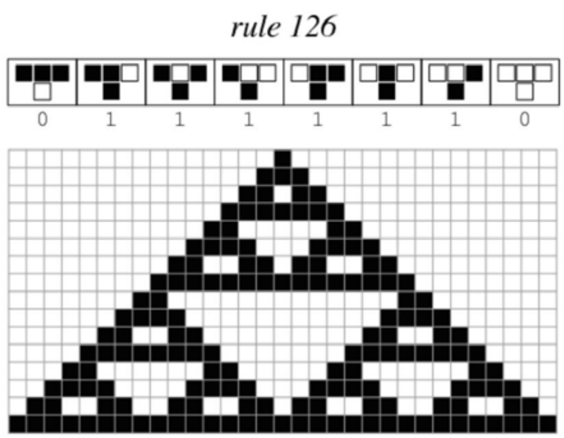


Figure 37: Wolfram Rule 126

[13] Wolfram, *New Kind of Science*.

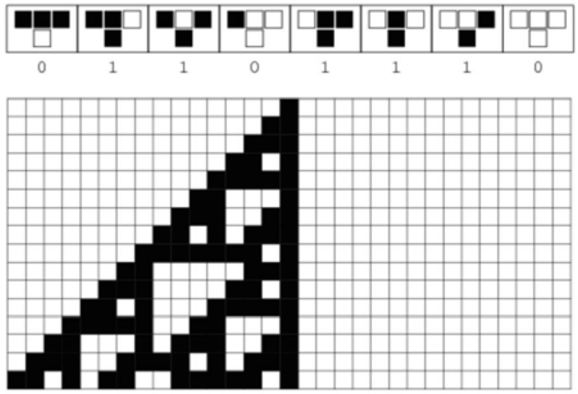


Figure 38: Wolfram Rule 110

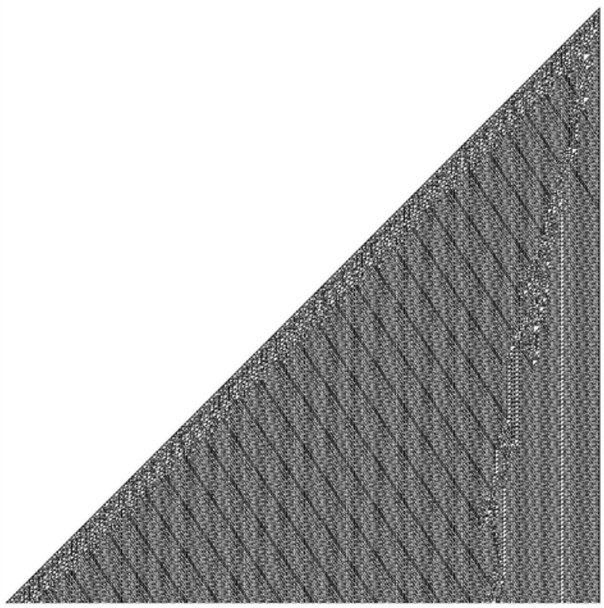


Figure 39: Wolfram Rule 110

5. The HighLife Replicator

This section is a comparison of patterns of the self-replicating element in the 1D RD and a very similar pattern in the much more complicated environment of the two-dimensional cellular automaton called HighLife, a variant of John Horton Conway's *Game of Life*. In HighLife, the environment is a rectangular lattice and each square is regarded as having eight neighbors. We could analyze an orthodox RD with an alphabet that generalizes the 16-letter alphabet to a $256 = 2^8$ letter alphabet for this geometry. This analysis is a future project for us. HighLife uses a simple binary rule. Each square in the lattice is either occupied (by a marker) or it is unoccupied (unmarked). We say that a square has n neighbors (where n is between 0 and 8) if n of its neighboring squares are occupied. The rule for HighLife is that an occupied square will survive (remain occupied) only if it has two or three neighbors. Otherwise it will become unmarked ("die"). An unoccupied square will become occupied (be "born") if it has three or six neighbors. In HighLife, there is a remarkable, small configuration that can reproduce itself. It takes 12 steps for this replication process to take place. See Figures 40-52.

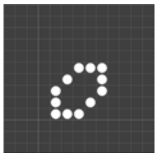


Figure 40: The HighLife Replicator

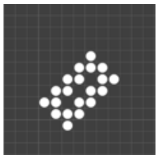


Figure 41: The HighLife Replicator

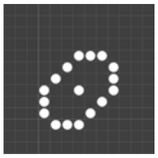


Figure 42: The HighLife Replicator

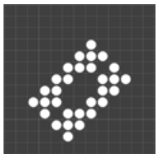


Figure 43: The HighLife Replicator

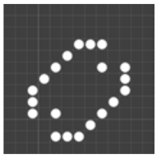


Figure 44: The HighLife Replicator

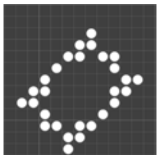


Figure 45: The HighLife Replicator

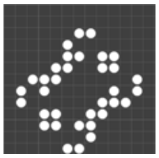


Figure 46: The HighLife Replicator

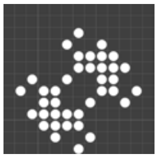


Figure 47: The HighLife Replicator

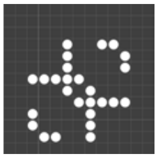


Figure 48: The HighLife Replicator

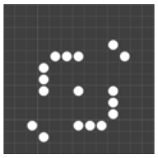


Figure 49: The HighLife Replicator

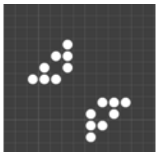


Figure 50: The HighLife Replicator

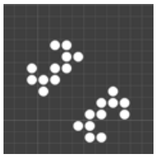


Figure 51: The HighLife Replicator

Figure 52: The HighLife Replicator

Quite remarkably, the pattern that these replicators follow is essentially the same as the pattern that is followed by the self-replicating element in the 1D RD. See Figures 53 to 60.

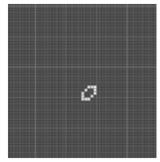


Figure 53: The 12-step HighLife replicator

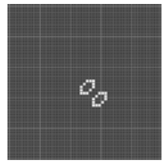


Figure 54: The 12-step HighLife replicator

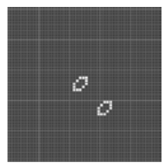


Figure 55: The 12-step HighLife replicator

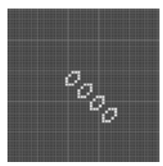


Figure 56: The 12-step HighLife replicator

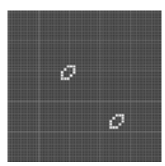


Figure 57: The 12-step HighLife replicator

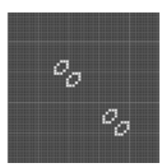


Figure 58: The 12-step HighLife replicator

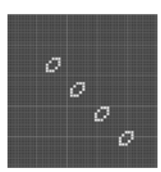


Figure 59: The 12-step HighLife replicator

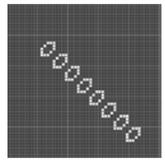


Figure 60: The 12-step HighLife replicator

6. RD and DNA

We begin this section with a review of material from the introduction to the paper. In this section, we describe one version RD process, and we show how it gives rise to a pattern of self-replication that is recognizable as a case of replication that we have called *DNA replication*.[14]

The rules for the RD process are very simple. We begin with an arbitrary, finite text string delimited by the character * at both ends. The RD process creates a new string from the given string by *describing the distinctions in the initial string.* Each character in the initial string is examined together with its left and right neighbors. Let LCR denote a character C with neighbors L and R. Then we replace C by a new character according to the following rules:

1. C → = if L = C and C = R (no distinction).
2. C → [if L ≠ C but C = R (distinction on the left).
3. C →] if L = C but C ≠ R (distinction on the right).
4. C → O if L ≠ C and C ≠ R (distinction on both the left and the right).
5. If C is adjacent to * change C to = (This is just a choice of boundary behavior).

See Figure 3 for the result of applying the RD process to a chosen text string.

In Figure 1, we showed the result of starting with a very simple text string. In this figure we do not print the character =, so that the resulting strings have empty space where this character would appear. As the reader can see, the string * ======]O[====== * has a long sequence of transformations under the RD process. The pattern]O[is replicated by the sequence below.

1 …=======]O[=======…

2 …======]OOO[=======…

3 …=====]O[=]O[======…

Remarkably, this self-replication has the same pattern as an abstract description of DNA replication. We explain this below in a separate section.

[14] Kauffman, “Biologic”; Kauffman, “Self-Reference.”

6.1 *A Quick Review of the Pattern of DNA Replication*

DNA consists of two strands of base-pairs wound helically around a phosphate backbone. It is customary to call one of these strands the Watson strand and the other the Crick strand. Abstractly, we can write

$$\mathrm{DNA} = < W \mid C >$$

to symbolize the binding of the two strands into the single DNA duplex. Replication occurs via the separation of the two strands via polymerase enzyme. This separation occurs locally and propagates. Local sectors of separation can amalgamate into larger pieces of separation as well. Once the strands are separated, the environment of the cell can provide each with complementary bases to form the base pairs of new duplex DNAs. Each strand, separated *in vivo*, finds its complement being built naturally in the environment. This picture ignores the well-known topological difficulties present to the actual separation of the daughter strands (see Figure 61). In this figure, we give some hints about the topological complexities that are not discussed here. Biologists discovered enzymes that cut and reconnect strands of DNA, resulting in the release of topological linking that would otherwise obstruct the separation of the newly produced strands of DNA. All this is subject to another discussion of its relationship with RD concepts.

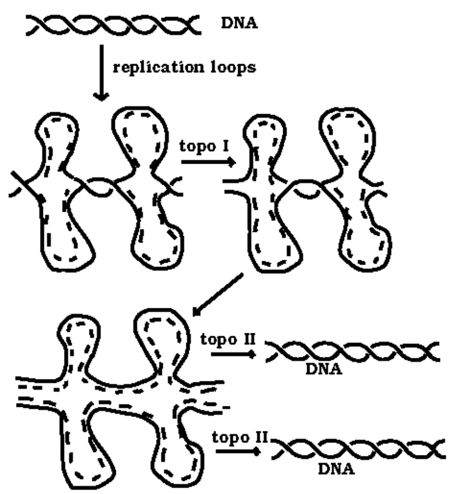


Figure 61: DNA Replication

The base pairs in the DNA sequence are AT (Adenine and Thymine) and GC (Guanine and Cytosine). Thus if

$$< W \mid = < \ldots \mathrm{TTAGAATAGGTACGCG} \ldots \mid$$

then

$$\mid C > = \mid \ldots \mathrm{AATCTTATCCATGCGC} \ldots >.$$

Symbolically we can oversimplify the whole process as

$$< W \,|\, + E \rightarrow < W \,|\, C > = \text{DNA}$$

$$E + |\, C > \rightarrow < W \,|\, C > = \text{DNA}$$

$$< W \,|\, C > \rightarrow < W \,|\, + E + |\, C > = < W \,|\, C >< W \,|\, C >$$

Either half of the DNA can, with the help of the environment, become a full DNA. We can let $E \rightarrow |\, C > < W \,|$ be a symbol for the process by which the environment supplies the complementary base pairs AG, TC to the Watson and Crick strands. In this oversimplification, we have cartooned the environment as though it contained an already-waiting strand $|\, C >$ to pair with $< W \,|$ and an already-waiting strand $< W \,|$ to pair with $|\, C >$.

In fact, it is the opened strands themselves that command the appearance of their mates. They conjure up their mates from the chemical soup of the environment.

The environment E is an identity element in this algebra of cellular interaction. That is, E is always in the background and can be allowed to appear spontaneously in the cleft between Watson and Crick:

$$< W \,|\, C > \rightarrow < W \,|\,|\, C > \rightarrow < W \,|\, E \,|\, C >$$

$$\rightarrow < W \,|\,|\, C >< W \,|\,|\, C > \rightarrow < W \,|\, C >< W \,|\, C >.$$

This is the formalism of DNA replication.

We are now in a position to compare the formalism of the DNA replication with the RD replication.

1 …=======]O[=======…

2 …======]OOO[=======…

3 …=====]O[=]O[======…

In the RD replication, we start with]O[in its RD environment. Matters of distinction of this entity from its surroundings lead to the production of]OOO[, and then we see that the identity of the internal O with its neighbors leads to the splitting]O[=]O[. There is no question that the basis of this replication is not the same as the DNA replication, but thematically, the two patterns are certainly related. The RD pattern is at a different level than the DNA pattern. In the RD replication, that environment for the symbol string is the larger symbol string. Thus it is only in the eyes of the observer of the RD that the entity]O[is distinguished and is seen as an actor against the background of declarations of identity …========…. These declarations of identity are indeed equal to one another and so form an invariant background or void from which patterns arise in the presence of any difference. This is, in fact how our entity came into being.

…AAAAAAAAAAAAABAAAAAAAAAAAAA…

…=============]O[==============…

Our entity]O[is the first description of sameness on left, difference in middle, sameness on the right. The left and right icons] and [form a carapace for the indicator of difference O. Thus a bare difference of B from its equal neighbors A evolves by description, at once into a *proto-cell* with a carapace. It is this protocell that then undergoes mitosis in the next two rounds of description. The cell-division or mitosis is enabled by the production of new carapace (]OOO[→]O[=]O[) from within the cell. It is important to note that this production does not come from an inner mechanism of the cell, but rather from the global recursive/descriptive situation of these entities in the entire line of the RD structure. It is the influence of the surrounding void that makes all this happen in the course of recursive description and distinction. It is a fortuitous accident of working in one dimension that the carapace is seen in a left portion paired with a right portion, analogous to the two strands of the DNA. *At this condensed creation scenario, we find that the patterns of DNA replication, cell formation, and mitosis all appear at once in the first few steps away from a marking (B) in the void (of repeated As).*

For DNA replication, we can interpret the correspondence as:

1.] = Watson, [= Crick, O = backbone or binding.
2. RD action results in the opening of the backbone so that binding O is replaced by environment OOO.
3. RD action relative to the environment results in the placement of a new Watson and a new Crick. So we have the self-replication of]O[.

Note that there is another level at which we can think about this! Regard] and [as cell walls. Then we are witnessing not DNA reproduction, but *mitosis* itself! The little fellow]O[is a cell and we are watching how it reproduces in the line environment ============ of the void where there are no distinctions. The reader should now look again at Figure 3 and note the many appearances and interactions related to this elementary cell.

Of course the interpretations of backbone, strand, environment, and cell are different from what happens in the biology, but it is very interesting that the basic principles are similar.

Note how we get …===]OOOOO… goes to …==]O[===… So actually the whole environment flips here. But it is contained in the above scenario. Everything that happens in RD is non-local, since a single event affects the whole string.

Perhaps it is clear to the reader that RD in the sense of this section is a potentially explosive topic that will grow to influence all the aspects of biology and computing. We believe that this is the case. The principle of [distinction/description in recursive process] applies at all levels of biology, cognition, information science, and computing.

7. Maturana, Uribe, and Varela and the Game of Life

Some examples from cellular automata clarify many of the issues about replication and the relationship of logic and biology. Here is an example due to Maturana, Uribe, and Varela.[15] The ambient space is two dimensional and in it there are "molecules" consisting of "segments" and "disks" (the catalysts; see Figure 62). There is a minimum distance between the segments and the disks (one can place them on a discrete lattice in the plane). And "bonds" can form with a probability of creation and a probability of decay between segment molecules with minimal spacing. There are two types of molecules: "substrate" (the segments) and "catalysts" (the disks). The catalysts are not susceptible to bonding, but their presence (within say three minimal step lengths) enhances the probability of bonding and decreases the probability of decay. Molecules that are not bonded move about the lattice (one lattice link at a time) with a probability of motion. In the beginning, there is a randomly placed soup of molecules with a high percentage of substrate and a smaller percentage of catalysts. What will happen over the course of time?

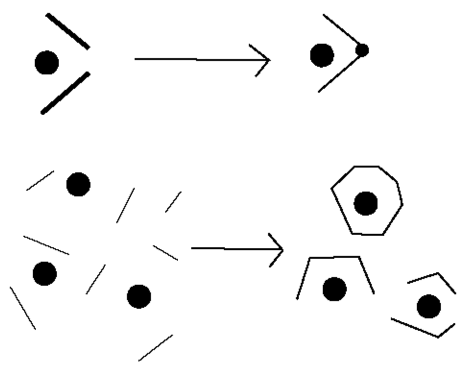


Figure 62: Proto-Cells of Maturana, Uribe, and Varela

In the course of time, the catalysts (which are basically separate from one another due to lack of bonding) become surrounded by circular forms of bonded or partially bonded substrate. A distinction (in the eyes of the observer) between inside (near the catalyst) and outside (far from a given catalyst) has spontaneously arisen through the "chemical rules." Each catalyst has become surrounded by a proto-cell. No higher organism has formed here, but there is a hint of the possibility of higher levels of organization arising from a simple set of rules of interaction. *The system is not programmed to make the proto-cells.* They arise spontaneously in the evolution of the structure over time.

8. Conway Life

One might imagine that organisms could be induced to arise as the evolutionary behavior of formal systems. There are difficulties, not the least of which is that there are

[15] H. R. Maturana, R. Uribe, and F. G. Varela, "Autopoesis: The Organization of Living Systems, Its Characterization and a Model," *Biosystems* 5 (1974): 7-13. See also F. J. Varela, *Principles of Biological Autonomy* (New York: North Holland Press, 1979) for a global treatment of related issues.

nearly always structures in such systems whose probability of spontaneous emergence is vanishingly small. A good example is given by another automaton – John H. Conway's Game of Life. In Life, the cells appear and disappear as marked squares in a rectangular planar grid. A newly marked cell is said to be born. An unmarked cell is dead. A cell dies when it goes from the marked to the unmarked state. A marked cell survives if it does not become unmarked in a given time step. According to the rules of Life, an unmarked cell is born if and only if it has three neighbors. A marked cell survives if it has either two or three neighbors. All cells in the lattice are updated in a single time step. The Life automaton is one of many automata of this type and indeed it is a fascinating exercise to vary the rules and watch a panoply of different behaviors.

For this discussion, we concentrate on some particular features. There is a configuration in Life called a glider (see Figure 63), which illustrates a series of gliders going diagonally from left to right down the Life lattice, as well as a glider gun (discussed below) that has produced them. The glider consists of five cells in one of two basic configurations. Each of these configurations produces the other (with a change in orientation). After four steps, the glider reproduces itself in form, but shifted in space. Gliders appear as moving entities in the temporality of the Life board. The glider is a complex entity that arises naturally from a small random selection of marked cells on the Life board. Thus the glider is a naturally occurring entity, just like the proto-cell in the Maturana-Uribe-Varela automaton.

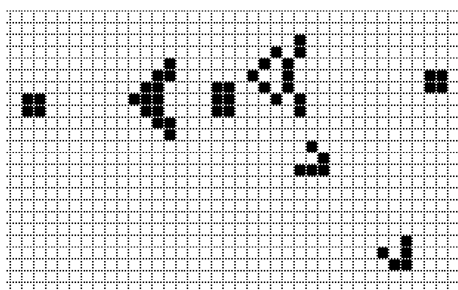


Figure 63: Glider gun and gliders

But Life contains potentially much more complex phenomena. For example, there is the glider gun (see Figure 63), which perpetually creates new gliders. The gun was invented by the Gosper Group, a group of researchers at MIT in the 1970s. It is highly unlikely that a gun would appear spontaneously in the Life board. Of course, there is a tiny probability of this, but we would guess that the chances of the appearance of the glider gun by random selection or evolution from a random state is similar to the probability of all the air in the room collecting in one corner. Nevertheless, the gun is a natural design based on forms and patterns that do appear spontaneously on small Life boards. The glider gun emerged through the coupling of the power of human cognition and the automatic behavior of a mechanized formal system.

Cognition is, in fact, an attribute of our biological system at an appropriately high level of organization. Cognition itself looks as improbable as the glider gun! Do patterns as complex as cognition or the glider gun arise spontaneously in an appropriate biological context?

There is a middle ground. If one examines cellular automata of a given type and varies the rule set randomly rather than varying the initial conditions for a given automaton, then a very wide variety of phenomena will present themselves. In the case of molecular biology at the level of the DNA there is exactly this possibility of varying the rules, in the sense of varying the sequences in the genetic code. So it is possible at this level to produce a wide range of remarkable complex systems.

9. Other Forms of Replication

Other forms of self-replication are quite revealing. For example, one might point out that a stick can be made to reproduce by breaking it into two pieces. This may seem satisfactory on the first break, but the breaking cannot be continued indefinitely. In mathematics, on the other hand, we can divide an interval into two intervals and continue this process ad infinitum. For a self-replication to have meaning in the physical or biological realm, there must be a genuine repetition of structure from original to copy. At the very least, the interval should grow to twice its size before it divides (or the parts should have the capacity to grow independently).

A clever automaton, due to Chris Langton, takes the initial form of a square in the plane. The square extrudes an edge that grows to one edge length and a little more, turns by ninety degrees, grows one edge length, turns by ninety degrees grows one edge length, turns by ninety degrees and when it grows enough to collide with the original extruded edge, cuts itself off to form a new adjacent square, thereby reproducing itself. This scenario is repeated as often as possible, producing a growing cellular lattice (see Figure 64).

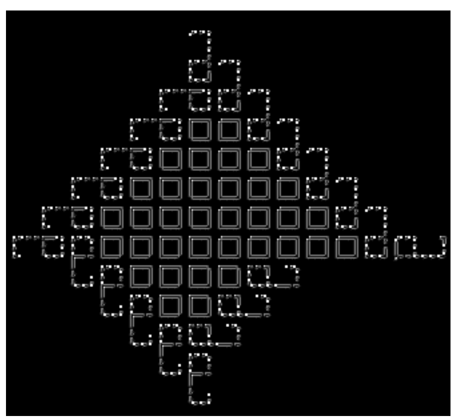


Figure 64: Langton’s automaton

The replications that happen in automata such as Conway’s Life are all really instances of periodicity of a function under iteration. The glider is an example where the Life game function L applied to an initial condition G yields $L^5(G) = t(G)$ where t is a rigid motion of the plane. Other intriguing examples of this phenomenon occur. For example, the initial condition D for Life shown in Figure 65 has the property that $L^{48}(D) = s(D) + B$ where s is a rigid motion of the plane and s(D) and the residue B are disjoint sets of marked squares in the lattice of the game. D itself is a small configuration of eight marked squares fitting into a rectangle of size 4 by 6. Thus D has a probability of 1/735471 of being chosen at random as eight points from 24 points.

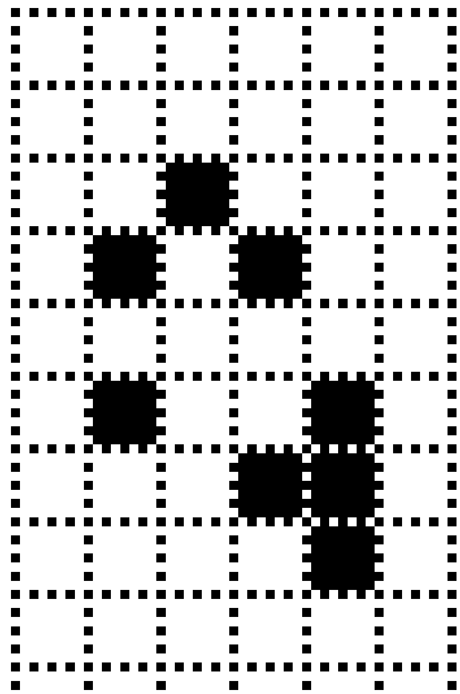


Figure 65: Condition D with geometric period 48

Should we regard self-replication as simply an instance of periodicity under iteration? Perhaps, but the details are more interesting in a direct view. The glider gun in Life is a structure GUN such that L^{30}(GUN) = GUN + GLIDER. Further iterations move the disjoint glider away from the gun so that it can continue to operate as an initial condition for L in the same way. A closer look shows that the glider gun is fundamentally composed of two parts P and Q such that L^{10}(Q) is a version of P and some residue, and such that L^{15}(P) = P* + B, where B is a rectangular block, and P* is a mirror image of P, while L^{15}(Q) = Q* + B' where B' is a small non-rectangular residue. See Figure 66 for an illustration showing the parts P and Q (left and right) flanked by small blocks that form the ends of the gun. One also finds that L^{15}(B + Q*) = GLIDER + Q + Residue. This is the internal mechanism by which the glider gun produces the glider.

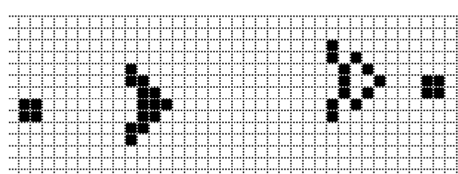


Figure 66: P (left) and Q (right) compose the glider gun

The extra blocks at either end of the glider gun act to absorb the residues that are produced by the iterations. Thus the end blocks are catalysts that promote the action of the gun. Schematically the glider production goes as follows:

$$P + Q \rightarrow P^* + B + Q^*$$

$$B + Q^* \rightarrow GLIDER + Q$$

whence

$$P + Q \rightarrow P^* + B + Q^* \rightarrow P + GLIDER + Q = P + Q + GLIDER.$$

The last equality symbolizes the fact that the glider is an autonomous entity no longer involved in the structure of P and Q. It is interesting that Q is a spatially and time shifted version of P. Thus P and Q are really copies of each other in an analogy to the structural relationship of the Watson and Crick strands of the DNA. The remaining part of the analogy is the way the catalytic rectangles at the ends of the glider gun act to keep the residue productions from interfering with the production process. This is analogous to the enzyme action of the topoisomerase in the DNA.

The point about this symbolic or symbiological analysis is that it enables us to take an analytical look at the structure of different replication scenarios for comparison and for insight.

There are a number of variants of Conway Life. We have earlier in this paper discussed HighLife and its self-replicator, whose pattern is a direct relative to the self-replicator in the 1D RD. Kauffman discussed another variant of Conway Life[16] and denoted it by the name 7-Life in that paper. The generative rule for 7-Life is B37/S23, meaning that an empty square gives birth to a marked square if it has either three neighbors or seven neighbors, and a marked square survives to the next generation if it has either two or three neighbors. Conway Life is defined by the distinction B3/S23. In Conway Life, one has gliders that occur naturally and we have discussed the glider gun that emerged from a design interaction with computer experiments using Conway Life. However, 7-Life behaves differently from Conway Life. There are still naturally occurring gliders, but relatively small initial configurations tend to behave dynamically, interacting via the gliders to produce self-sustaining, slowly growing configurations. These configurations can eventually give birth to more complex self-reproducing entities.[17] The entity that emerges, usually after thousands of iterations, is more complex (a pair of mirror-imaged configurations) than the glider, but by our experience, not so improbable as never to emerge! This leads to the question of the possibility and probability of the emergence of complex structures, analogous to biological structures, in the forward history of an RD automaton. We mention the cases of non-orthodox RD and experiments of this kind since structurally, all these automata do operate recursively on the basis of distinctions made at each step. The variants of Conway Life and the Wolfram automata are all very simple instances of RD where the basic language is binary and there is only one distinction made at each step.

10. Laws of Form

In this section, we discuss a formalism of G. Spencer-Brown in his book *Laws of Form*,[18] which is often called the calculus of indications. This calculus is a study of mathematical foundations with a topological notation based on one symbol, the mark

[16] Kauffman, “Reflexivity and Eigenform.”
[17] Ibid.
[18] G. Spencer-Brown, *Laws of Form* (New York: Julian Press, 1969).

$$\overline{\;\;}|.$$

This single symbol represents a distinction between its own inside and outside. The mark is seen as making a distinction, and the calculus of indications is a calculus of distinctions, where the mark refers to the act of distinction. The mark is self-referential and refers to its own action and to the distinction that is made by the mark itself. Spencer-Brown is quite explicit about this identification of action and naming in the conception of the mark, and by the end of the book he reminds the reader that "the mark and the observer are, in the form, identical." We make this discussion here because it is important to trace the origins of the idea of distinction that is so central to the present paper.

The concept of distinction as used in Laws of Form is very close to that used implicitly in set theoretic mathematics. There the fundamental distinction is represented by set brackets (the act of collecting into a set) and the empty set { } is the first distinction.

In the calculus of indications, the mark can interact with itself in two possible ways. The resulting formalism becomes a version of Boolean arithmetic, but fundamentally simpler than the usual Boolean arithmetic of 0 and 1 with its two binary operations and one unary operation (negation).

Remarkably, the calculus of indications provides a context in which we can say exactly that a certain logical particle, the mark, can act as negation *and* can interact with itself to produce itself.

The mathematics in Laws of Form begins with two laws of transformation about these two basic expressions. Symbolically, these laws are:

1. Calling $\overline{\;\;}|\,\overline{\;\;}| = \overline{\;\;}|$
2. Crossing $\overline{\overline{\;\;}|}| =$

The equals sign denotes a replacement step that can be performed on instances of these patterns (two empty marks that are adjacent or one mark surrounding an empty mark). In the first of these equations, two adjacent marks condense to a single mark, or a single mark expands to form two adjacent marks. In the second equation, two marks, one inside the other, disappear to form the unmarked state indicated by nothing at all. That is, two nested marks can be replaced by an empty word in this formal system. Alternatively, the unmarked state can be replaced by two nested marks. These equations give rise to a natural calculus, and the mathematics can begin. For example, *any expression can be reduced uniquely to either the marked or the unmarked state.* The following example illustrates the method:

$$\overline{\overline{\overline{\overline{\;\;}|\,\overline{\;\;}|}|\,\overline{\;\;}|}|\,\overline{\;\;}|}|\,\overline{\;\;}| = \overline{\overline{\overline{\overline{\;\;}|}|\,\overline{\;\;}|}|\,\overline{\;\;}|}|\,\overline{\;\;}| = \overline{\overline{\overline{\;\;}|}|\,\overline{\;\;}|}|\,\overline{\;\;}|$$

$$= \overline{\overline{\;\;}|}|\,\overline{\;\;}| = \overline{\;\;}|$$

The general method for reduction is to locate marks that are at the deepest places in the expression (depth is defined by counting the number of inward crossings of boundaries needed to reach the given mark). Such a deepest mark must be empty and it is either surrounded by another mark, or it is adjacent to an empty mark. In either case, a reduction can be performed by either calling or crossing.

Laws of Form begins with the following statement. “We take as given the idea of a distinction and the idea of an indication, and that it is not possible to make an indication without drawing a distinction. We take therefore the form of distinction for the form.” Then the author makes the following two statements (laws):

1. The value of a call made again is the value of the call.
2. The value of a crossing made again is not the value of the crossing.

The two symbolic equations above correspond to these statements. First, examine the law of calling. It says that the value of a repeated name is the value of the name. In the equation

$$\urcorner\,\urcorner = \urcorner$$

one can view either mark as the name of the state indicated by the outside of the other mark. In the other equation

$$\overline{\urcorner\,}\!\!\mid \;=$$

the state indicated by the outside of a mark is the state obtained by crossing from the state indicated on the inside of the mark. Since the marked state is indicated on the inside, the outside must indicate the unmarked state. The Law of Crossing indicates how opposite forms can fit into one another and vanish into nothing, or how nothing can produce opposite and distinct forms that fit one another, hand in glove. The same interpretation yields the equation

$$\urcorner = \urcorner$$

where the left-hand side is seen as an instruction to cross from the unmarked state, and the right hand side is seen as an indicator of the marked state. The mark carries a double meaning. It can be seen as an operator, transforming the state on its inside to a different state on its outside, and it can be seen as the name of the marked state. That combination of meanings is compatible in this interpretation.

From the calculus of indications, one moves to algebra. Thus

$$\overline{\overline{A\,}\!\!\mid\,}\!\!\mid$$

stands for the two possibilities

$$\overline{\overline{\overline{}|}|}| = \overline{}| \longleftrightarrow A = \overline{}|$$

$$\overline{\overline{}|}| = \quad \longleftrightarrow A =$$

In all cases we have

$$\overline{\overline{\mathrm{A}}|}| = A$$

By the time we articulate the algebra, the mark can take the role of a unary operator

$$A \longrightarrow \overline{\mathrm{A}}|$$

But it retains its role as an element in the algebra. Thus begins algebra with respect to this non-numerical arithmetic of forms. The primary algebra that emerges is a subtle precursor to Boolean algebra. One can translate back and forth between elementary logic and primary algebra:

1. $\overline{}| \longleftrightarrow T$
2. $\overline{\overline{}|}| \longleftrightarrow F$
3. $\overline{\mathrm{A}}| \longleftrightarrow \sim A$
4. $AB \longleftrightarrow A \vee B$
5. $\overline{\overline{\mathrm{A}}|\,\overline{\mathrm{B}}|}| \longleftrightarrow A \wedge B$
6. $\overline{\mathrm{A}}|B \longleftrightarrow A \Rightarrow B$

The calculus of indications and the primary algebra form an efficient system for working with basic symbolic logic.

By reformulating basic symbolic logic in terms of the calculus of indications, we have a ground in which negation is represented by the mark *and* the mark is also interpreted as a value (a truth value for logic) and these two interpretations are compatible with one another in the formalism. At this point the reader can appreciate what has been done if he or she returns to the usual form of symbolic logic. In that form we see that

$$\sim\sim X = X$$

for all logical objects (propositions or elements of the logical algebra) X. We can summarize this by writing

$$\sim\sim =$$

as a symbolic statement that is outside the logical formalism. Furthermore, one is committed to the interpretation of negation as an operator and not as an operand. The calculus of indications provides a formalism where the mark (the analog of negation in that domain) is both a value and an object, and so can act on itself in more than one way.

The mark as linguistic particle is its own anti-particle. It is exactly at this point that physics meets logical epistemology. Negation as logical entity is its own anti-particle. In our view, the world and the formalism we use to represent the world are not separate. The observer and the mark are (formally) identical. A path is opened between logic and physics.

The visual iconics that create via the half-boxes of the calculus of indications a model for the mark as logical particle can also be seen in terms of cobordisms of surfaces (see Figure 67). There the boxes have become circles and the interactions of the circles have been displayed as evolutions in an extra dimension, tracing out surfaces in three dimensions. The condensation of two circles to one is a simple cobordism between two circles and a single circle. The cancellation of two circles that are concentric can be seen as the right-hand lower cobordism in this figure with a level having a continuum of critical points where the two circles cancel. A simpler cobordism is illustrated above on the right where the two circles are not concentric, but nevertheless are cobordant to the empty circle. Another way of putting this is that two topological closed strings can interact by cobordism to produce a single string or to cancel one another. Thus, a simple circle can be a topological model for the mark, for the fundamental distinction.

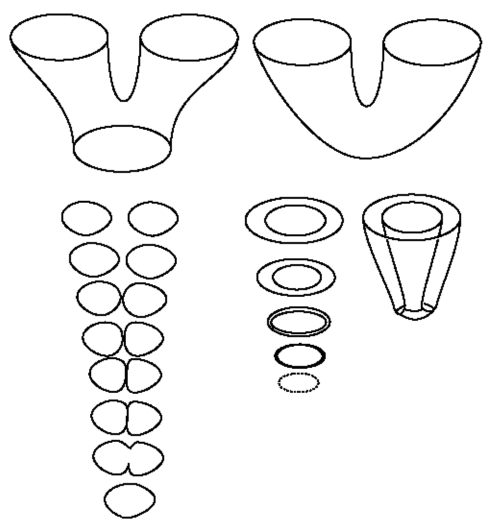


Figure 67: Calling, crossing, and cobordism

We are now in a position to discuss the relationship between logic and quantum mechanics. We go below Boolean logic to the calculus of indications, to the ground of distinctions based in the phenomenology of distinction arising with the emergence of concept and percept together, in the emergence of a universe in an act of perception.

Here we find that the distinction itself is a logical particle that can interact with itself to produce itself, but can also interact with itself to annihilate itself. The fundamental state is a superposition of these two possibilities for distinction. We are poised between affirmation of presence and the fall into an absence that we cannot know. This superposition is likely not yet linear in the sense of the simple model of quantum theory. Nevertheless, it is at this source, the place of arising and disappearing of awareness, that we come close to the quantum world in our own experience. As always, this experience is known to us in ways more intimate than the reports of laboratory experiments. It is the uniqueness of every experience, of every distinction. There can be no other one. There is only this and this and this yet again.

Nevertheless, one can go on and consider quantum states related to the aforementioned logical particle. Crossing this boundary into quantum theory proper, one finds that topology and physics come together in this realm, and there is a complex possibility of much new physics to come and a new basis for quantum computing.[19] It will take more thought and a sequel to this paper to begin to sort out the relationships between quantum theory and RD at the level of this form of epistemology.

Remark. In Laws of Form we can express XOR(A, B) = A^B = B^A by the formula

$$A^B = B^A = \overline{\overline{\mathrm{A}\rvert}\,\mathrm{B}\rvert}\;\overline{\mathrm{A}\;\overline{B\rvert}\rvert}$$

Note that if B is marked, then

$$A^{\overline{}\rvert} = \overline{\mathrm{A}\rvert}$$

Thus the operation of XOR is the action of the mark itself. We can regard diagrammatic circuits such as we used in Figure 6 as applications of the mark in the form of the XOR operation above. In this way, the apparently awareness-dependent operations of the Laws of Form shift to the automatic discrimination capabilities of computer circuits and the forms of RD can be seen as written in the language of the calculus of indications. These points of view inform each other circularly.

11. Commentary

Here is a collection of remarks and insights into RD that come from conversations between the authors of this paper over a number of years.

1. Joel: When distinction-making is applied to a pattern there is a new pattern that is comprised of the variety of distinctions recorded. Thus, a new pass of distinction-making can be applied to the pattern of distinctions, and this kind of a process can repeat itself recursively, indefinitely.

[19] For further details, see Louis H. Kauffman. “Knot Logic and Topological Quantum Computing with Majorana Fermions,” in *Logic and Algebraic Structures in Quantum Computing and Information: Lecture Notes in Logic*, ed. J. Chubb, Ali Eskandarian, and V. Harizanov (Cambridge: Cambridge University Press, 2016), 223-336.

2. Joel: I had made a discovery (mathematical in nature) of processes of RD (which is not patentable *per se*), and then invented a physical embodiment that performs these processes.

3. Joel: The sensing of gradients (chemical concentrations, nutrients, etc.) in bacteria is well established and demonstrable. These are elements of distinction-making at very primitive levels. It is much harder to demonstrate recursive distinction-making in bacteria, because these are more abstract operations. It can be done however with live neuron circuits, and about 250K separate us now from results of such a demo.

   Eshel Ben-Jacob proposed that recursive distinction-making may be easier to demonstrate in genetic/immunological systems and it would also be much cheaper than the work planned with neurons. I am waiting for more details. At any rate, Eshel's program is all interrelated, with recursive distinction-making being a unifying theme.

4. Joel: I tend to think in terms of sensory-driven cognition that is constructed bottom-up, beginning with Stage 1 – sensory distinctions; and proceeds to Stage 2 – indefinite recursion that starts out from Stage 1 and builds up successive layers of distinctions-of-distinctions. It is unlikely that these two low-level stages involve awareness. A working hypothesis is that some sort of awareness emerges from the primitive Stages 1 and 2 towards a level that you identified as Type 1. So, basically I tend to think of your Type 1 as an epiphenomenon that arises from Stages 1 and 2. [Type 1 for LK is a distinction that comes simultaneously with an awareness of that distinction.] I believe (actually I have shown) that Stages 1 and 2 are mechanizable. A missing link, of course, is the transition from Stages 1 and 2 to your Type 1. I am very sympathetic to constructivist dispositions and the place of human beings in the order of things. I agree that thought thinking itself is all we have got … but I see no contradiction in proposing that thought processes have their ultimate genesis in pre-cognitive and pre-aware primitive processes of sensory-driven RD.

5. Joel: Spencer-Brown has been very seductive to a lot of people and rightfully so. For most of us, drawing a distinction is a cognitive act that is performed by a full-blown human being. Spencer-Brown, of course, represents a distinction by some sketching of circles on a piece of paper by a human. I don't object to this! That's how much of mathematics is done. Scribbling of some symbols, sometimes in reference to some drawings of geometric or topological configurations. But doubts linger. Is it possible to entertain a situation where distinctions are drawn by acts that are short of being cognitive? And if this is possible, where is the observer, the self? And what constitutes the other? What will happen to the expected dynamics of "I and Thou"? Will there emerge a "becoming"? Becoming of what? It seems utterly futile to concoct a scenario of distinction-making at a level that is well below a cognizing person. (And what's left of constructivism if the cognizing person is dissolved to his sensory modalities?) [LK: Note that Spencer-Brown never discusses how distinctions arise but always discusses distinctions that are accompanied by an awareness or an observer.]

Well, the thing is this. Sensory modalities, all of them, must make local distinctions in certain features (e.g., intensities) in signals that impinge on them. It has been studied in great detail in visual perception, beginning with the retina. Photoreceptors in the retina make local distinctions of light intensities that impinge on the retina. (Absent this, capacity for local distinctions amounts to blindness.) This local distinction-making is accomplished by comparisons that ultimately cause firing/non-firing. These processes involve certain physiological/biochemical processes, in conjunction with massive neural circuits. The above type activity is clearly pre-cognitive, involuntary, and (with sufficient abstraction) can be accomplished by computing machines as well.

The essence of my patent document is RD (in one-dimension; but it is motivated directly by RD in 2D, which operates on 2D digital imager; 2D RD is abstracted from local distinction-making at the retinal level, as worked out by Weisel and Hubel in the early 1960s).

I recently sketched for the history of my ideas (beginning in the early 1960s) and how these are embedded in the patent document, including the basis for *fantomarks* and their streaks.

I think that the singular contribution of my particular RD processes is operationalizing the process of recursion on distinction-making. For it gives precise and detailed trace of what it entails, including an emergent dialectics, circularity, and so on.

To be sure, other people have talked about recursive distinction-on-distinction (notably Maturana, in the context of his much higher-level “languaging”), but it should be clear that my RD is at a precognitive level, is mechanizable, and affords a thorough examination of its emergent properties.

6. Joel: I noticed that thing – the hypothetical distinction (or contingent distinction) that hasn’t actually been made. It exemplifies the potency of distinction, even if not acted upon. These are the wonders of distinction, actual, virtual, potential, contingent, and hidden, to name only a few types. Now, when these are compounded via recursion – watch out!

7. Joel: I have no objection to make a (provisional) distinction between the kind of distinction in RD automata and the Maturana and Varela kind of distinction. In itself, this act of distinction between two distinctions is a good example of what RD automata typically do. I think that, in the end, we’ll mutually discover that the distinction between the two kinds of distinctions will gradually dissolve.

   Here is a succinct description of the roles of distinction in RD automata: In RD automata, we have two basic elements that involve notions of distinction.

   1. An element of distinction-making. This element involves acts of distinguishing (verb) and is a process.

2. The results of distinctioning are a collection of distinctions; where a distinction is a product, object (noun).

Usually these objects form a pattern of distinctions (the pattern as a whole is also an object) that is subject to further acts of distinctioning.

Thus process and products alternate, recursively, where both process and products involve notions that relate to distinction.

The process element involves distinction-making; and the product element is a pattern of objects, referred to as distinctions. (Each such distinction is a local, fragmentary boundary that records the result of prior acts of distinctioning.)

It is crucial to understand that the alternation between process/product is recursive and indefinite in duration; also, that such indefinite recursion is guaranteed to drive the process into circularity. This, as a whole, represents the notion of RD in RD automata. (It is called BIP in the patent.)

The RD automata model is motivated by natural vision. The initial stages are motivated by the retina, and the rest of the recursive process is postulated to take place in the lateral geniculate nucleus (LGN) and the visual cortex proper.

In recent years, some researchers in advanced techniques in neural circuits (not artificial neural nets, but rather actual, live neural tissue) have entertained the hypothesis that a certain version of RD automata takes place in normal brain tissue activity.

8. Joel: This is to systematize RD by dimension.

* 1D – This is the case that is documented in the patent. It was pre-dated by the 2D case. A neighborhood comprises three elements, where a central element has two neighbors. There are exactly four combinations of relationships between an element and its two neighbors, representable by four ideographs, as described in the patent.

* 2D – This is the case that relates to image processing; it goes back to 1964. A neighborhood (Moore neighborhood) is comprised of nine elements, where a central element has eight neighbors. There are exactly 256 combinations of relationships between a central element and its eight neighbors. These are representable by 256 ideographs.

The 2D case can be decomposed into a network of 1Ds. For comparison, John Conway's Game of Life is also run on a Moore neighborhood but has only two states (as compared to 256 [!] states in the Game of RD). The richness and complexity of Game of Life is well known. Imagine the complexity of this 2D RD game.

* 3D – A neighborhood is comprised of 27 elements, where a central element has 26 near neighbors. There are exactly 2^{26} (i.e., 67,108,864) combinations of

relationships between a central element and its 26 near-neighbors. Clearly, I didn't investigate this case. Instead, I retreated to the 0D case; see below.

* 0D – This is the case where RD starts with a single speck against the void. It yields the scenario of the baryon octet, as described elsewhere.

9. Joel and Lou: Your comment is interesting. There are RD processes that are uniquely in the purview of human observers. There are certain RD processes that can be performed by automata, and there may also be RD processes in nature. The challenge is to integrate all three types into an encompassing framework whose unifying theme is RD processing. As to experimenting with CA, there are obviously untold numbers of possible CA, some of which have extremely interesting behaviors. In RD we focus on a singular cellular automaton, the one CA whose rule is recursive distinction-making. Once we grasp that distinction-making is a unique operation (in regards to perception and cognition) we realize that we must focus on the particular class of RD automata, in preference to the other zillions of CA possibilities that are available for our consideration and entertainment. I submit that RD automata are the needles in the haystack of CA.

10. Lou: In programs that we design the initial automatic distinctions are distinctions that are put in by design. In the observation of such programs new distinctions arise for us, that can be used for further designs. But in nature, it is not obvious how those structures that we are calling distinction operators have arisen. We do not imagine that they occur by design. We do not imagine that they were ideas in the mind of a designer. I am very aware of this issue. as I have experimented at other levels with cellular automata and have seen how by varying rule structures one can find extraordinary recursive structures that one would never have imagined. Our relationship with our own constructions and with nature is complex.

11. Joel: Transdistinction operates on patterns of raw sensory data to produce a first pass of local distinction-making in such patterns. Further processing is relegated to higher centers in the nervous system. (For example, this is essentially what the retina does [in part] in vision.) This first pass is relatively easy to accomplish by computing devices. Thus, impairment in a sensory organ can be overcome by using such prosthetic devices. The next issue, of course, is how to connect the output of the prosthetic device to higher centers. In vision, for example, a connection needs to be done to the optic nerve, or directly to the lateral geniculate nucleus, from which the normal vision pathways would be followed to the visual cortex. Assuming that such devices will become reality, would it modify our notion of the observer? Namely, a human observer so equipped would initiate his or her observation by an automatic device that does distinction-making. So, there you have it – a hybrid of human/machine in a long sequence of distinction-making; some automatic and some human-based.

12. Joel: Yes, quids and quods seem to be generalized notions of containers/extainers. [Lou: Extainers have the formalism E = >< while containers have the formalism C =

<>.] Extainers are open to interaction from the outside. Containers are closed forms not likely to interact. But note that

$$EE = ><>< = > C <$$

and

$$CC = <><> = < E >.$$

Thus an extainer interacts with an extainer to produce a container, and a container interacts with a container to produce an extainer. We can distinguish between containers and extainers by allowing containers to move freely (commute) with other elements. Then

$$EE = > C < = C >< = CE$$

and we see that C can be the catalyst for self-replication. And if we regard the extainer as the environment, then the movement

$$<> \rightarrow < E >$$

can be seen as our earlier abstraction of the emergence of Watson and Crick strands from the environment. We obtain the self-replication of DNA type:

$$<> \rightarrow < E > \rightarrow <><>.$$ [20]

Inasmuch as quids and quods come about literally out of nowhere (they are byproducts of RD that operates on arbitrary initial unspecified things, including fantomarks), their natural algebra may be significant.

Quids and quods (discrete/continuous) are self-organized. They enter into an elaborate dance that is not choreographed by external manipulation. The dance has classical dialectical patterns.

Replication is part of the game. There are at least two types of replication:

1. For RD with fixed boundaries, there is guaranteed circularity. Thus a whole bunch of strings are periodically replicated. These happen to be 4-letter strings with certain complementarity properties. Close enough to DNA, but not quite the same.

2. For RD with shifting extainers (such as in the Baryon Octet scenario), there is replication of patterns via self-similarity in the trace. In effect, a basic pattern reappears periodically.

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[20] See Kauffman, "Iterants, Fermions, and Majorana Operators"; Kauffman, "Biologic"; and Kauffman, "Self-Reference, Biologic and the Structure of Reproduction" for more about extainers and containers.

All in all, I propose to consider the algebra of quids/quods (which extends your [Lou K.]) notions of containers/extainers) somewhere at the foundations of your marvelous edifice.

There is an example of this in Figure 2 on page 11 of my paper "Steganogramic Representation of the Baryon Octet in Cellular Automata." This is an RD that starts out with a first arbitrary distinction. Focus on lines 1 thru 8.

'0' is like your container that fuses <> together. It may contain at most one thing. There is a notion of extended container, written: < * … * >, which may contain a bunch of things. (It shows in Figure 2 as [==== … =]: C is an element of quids, and the extended container is a quod, as defined in the patent document.) Now, make the following substitutions in Fig. 2:

0 is C

] is >

[is <

= is *

Lines 1 thru 8 will look like this:

>C<

>CCC<

>C<*>C<

>CCCCCCC<

>C<*****>C<

>CCC<***>CCC<

>C<*>C<*>C<*>C<

>CCCCCCCCCCCCCCC<

and you can continue thru line 16 and beyond. Within that 16-line diagram you can identify 10 configurations that look like this:

>C<

CCC

<*>

Those 10 configurations are self-organized similarly to the Pythagorean Tetractys.[21] Those configurations allow us to uncover the configuration of the baryon octet that is embedded therein.[22]

Thus the physical interpretation of > and < are up and down quarks and * is a strange quark.

Let's recoup what we're doing. We start out with a first distinction and apply RD to it. We develop the trace of a cellular automaton that does RD. Within that trace we discover the Pythagorean Tetractys, within which we discover the eight particles of the baryon octet expressed in terms of their constituent quarks. Note: There ought to be a link to SU(3), which still eludes me.

13. Lou: Clearly we have just begun this study. There is much more to come.

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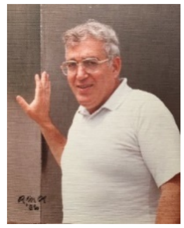
About the Authors: Louis H. Kauffman is Professor of Mathematical Physics and Cybernetics at the University of Illinois, Chicago. He has degrees from MIT and Princeton. He has 170 publications. He was the founding editor for the *Journal of Knot Theory and its Ramifications* and he writes a column entitled "Virtual Logic" for the journal *Cybernetics and Human-Knowing*. He was president of the American Society for Cybernetics from 2005-2008. He introduced and developed the Kauffman Polynomial. He was the recipient of the 2014 Norbert Wiener Award of the American Society for Cybernetics.

Joel D. Isaacson has pioneered in RD Cellular Automata since the 1960s. Recursive Distinctioning was rooted in studies relating to the analysis of digitized biomedical imagery. He utilized NASA's computing facilities at the Goddard Space Flight Center in Greenbelt, MD for the initial stages of this research. His research has been supported over the years by DARPA, SDIO, NASA, ONR, USDA and a good number of NIH institutes. He is Professor Emeritus of Computer Science, Southern Illinois University, and Principal Investigator for the IMI Corporation.

[21] Isaacson, "Steganogramic Representation," Figure 3, p. 12.
[22] Ibid., Figure 7, p. 16.



Editors' Notes: The Board of Directors of Kepler Space Institute (KSI) and the editors of the *Journal of Space Philosophy* take pride in providing the publication platform for Dr. Joel D. Isaacson and Dr. Louis H. Kauffman to inform the public on the current status of RD. That term is the scientific description of "Nature's Cosmic Intelligence" (Joel Isaacson, *Journal of Space Philosophy* 1, no. 1 [Fall 2012]: 8-16) that Dr. Isaacson discovered in 1964, Since that date he has been the lead scientist and scholar in researching this information stream phenomenon that Dr. Bernd Schmeikal – whose supporting paper is also in this Special Science Issue of the *Journal of Space Philosophy* – has called "a universal creative system." Dr. Isaacson described RD in April 2011 as "a finding that is advanced as a law of nature, perhaps on the par of gravity." Over the past two years, Dr. Louis Kauffman, one of America's most distinguished mathematicians and physicists, has worked intensively with Dr. Isaacson to create this latest scientific explanation for the world. For further information on RD, see homepages.math.uic.edu/~kauffman/RD.html. ***Bob Krone and Gordon Arthur.***