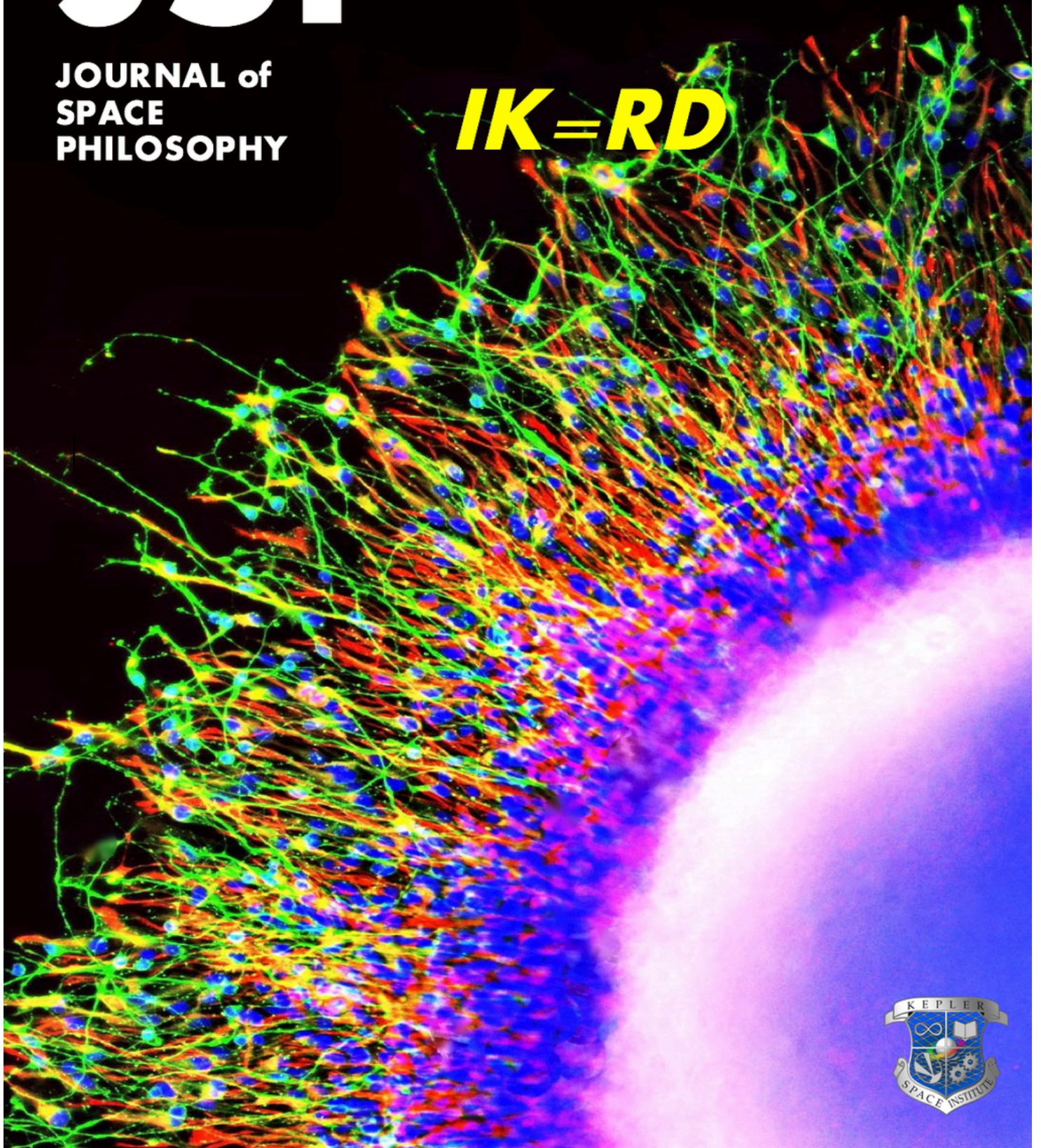


JSP

JOURNAL of
SPACE
PHILOSOPHY

Special Science Issue
Recursive Distinctioning

IK=RD





DEDICATION

By the Kepler Space Institute Board of Directors

This special science issue of the *Journal of Space Philosophy* is dedicated to Earth's space scientists, who have brought humanity, through their sciences, to the brink of the Space Epoch. That epoch will contain new worlds for humans holding the promise of improvements and survival for our species beyond our current imaginations. The feature article in this Spring 2016 issue of the *Journal of Space Philosophy*, "Recursive Distinctioning," by scientists Joel D. Isaacson and Louis H. Kauffman, is a major addition to this historic legacy.

This issue is also dedicated to the memory of Eshel Ben-Jacob whose recursive creativity has halted abruptly and whose ideas continue to dwell in these pages. The picture at the bottom of the page is of actual bacteria colonies that Eshel used to refer to as "bacteria art." He and Joel Isaacson made the tacit assumption that bacteria use RD for their intelligence and organization.





Preface

By Bob Krone and Gordon Arthur

This Special Science issue of the *Journal of Space Philosophy* is a catalyst for us to reflect on the five years we have been editing the Journal for you readers.

It was a surprise in 2012 that the global Space community had not created a journal of philosophy when most of the hard and soft sciences had done so long in the past. The *Journal* has given us in the Kepler Space Institute (KSI) a tangible academic product while we planned for the future. We did that planning without external funding, which meant that KSI Board and other members were funding their own participation in Space Conferences and our operations.

We decided to publish our KSI agreed Space philosophy in the first issue, Fall 2012. Its fundamentals are:

- veneration for life;
- within ethical civilizations;
- implemented by the Policy Sciences.

Over the past five years, we have found no reason to change those fundamentals. Many of the articles published since 2012 provide the details of that philosophy.

The research area we take great pride supporting is the ongoing discoveries and analysis of the Recursive Distinctioning (RD) natural intelligence feature discovered in 1964 by Dr. Joel Isaacson. Louis H. Kauffman, PhD, University of Illinois at Chicago, presented the latest scientific findings on RD at the National Space Society's International Space Development Conference in Puerto Rico, May 18-22, 2016 (ISDC-2016), using the feature paper in this issue, "Recursive Distinctioning," by Joel D. Isaacson and Louis H. Kauffman, as a basis for his remarks.

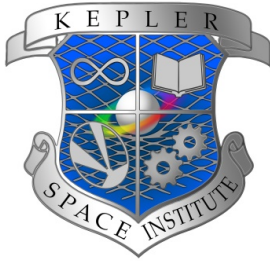
Recursive Distinctioning is fundamental to all perception, and, by extension, to cognition and intelligence. That finding is advanced as a law of nature, perhaps on a par with gravity. (Joel Isaacson, e-mail to Bob Krone, April 20, 2011)

This Special Science Issue is devoted exclusively to enhancing global awareness of Recursive Distinctioning and its implications for the future of humanity on Earth and in Space.

Journal of Space Philosophy 5, no. 1 (Spring 2016)

The original cover for this Special Science RD Issue was designed by Naté Sushereba and Joe Sobodowski, both members of the Kepler Space Institute Board of Directors

Bob Krone, PhD, Editor-in-Chief
Gordon Arthur, PhD, Associate Editor



JOURNAL OF SPACE PHILOSOPHY

Vol. 5, No. 1, Spring 2016

Special Science Issue

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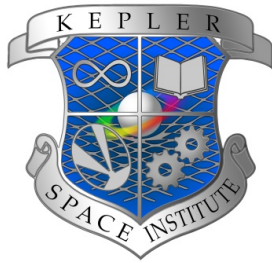
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Access to the Journal of Space Philosophy and free downloading of its articles is available at bobkrone.com/node/120.

Anyone on Earth or in Space may submit an article or Letter to the Editor to BobKrone@aol.com.



PRESS RELEASE

June 30, 2016

By Gordon Arthur

Kepler Space Institute has released the latest edition of the *Journal of Space Philosophy*. The Spring 2016 issue, its first Special Issue, focuses on the work of Joel Isaacson and Louis Kauffman on Recursive Distinctioning (RD). It begins with an introduction to the concepts of RD by Bob Krone, and then has two feature articles: “Recursive Distinctioning,” by Joel Isaacson and Louis Kauffman, and “Basic Intelligence Processing Space,” by Bernd Schmeikal.

Isaacson and Kauffman’s paper explains how very simple operations, such as substituting letters in a specialized alphabet for letters in a word or other symbols by drawing distinctions between different conditions, can lead to surprisingly complex, recurring patterns. They then explain how some of the most fundamental principles of physics, biology, and symbolic logic can be written and manipulated in this way. Bernd Schmeikal then applies this to certain problems in theoretical physics.

In the words of Joel Isaacson, RD “is fundamental to all perception, and, by extension, to cognition and intelligence. That finding is advanced as a law of nature, perhaps on a par with gravity” (e-mail to Bob Krone, April 20, 2011). In the word of Bob Krone, Editor-in-Chief of the *Journal of Space Philosophy*, this issue “will surely be the most important issue to date” (e-mail to Gordon Arthur, March 16, 2016).

Notes from the Chair

By Gordon Holder, VADM, US Navy (Ret), Kepler Space Institute Chairman of the Board

The Board of Directors, Staff, and Members of Kepler Space Institute (KSI) take special pride as this Spring 2016 issue of the *Journal of Space Philosophy* is published.

At the International Space Development Conference in Puerto Rico, May 18-22, 2016 (ISDC-2016) Louis H. Kauffman, PhD, University of Illinois at Chicago, presented the latest scientific findings on Nature's Cosmic Intelligence, scientifically identified as *Recursive Distinctioning* (RD). His primary reference was this Special Issue's feature paper, "Recursive Distinctioning," by Joel D. Isaacson and Louis H. Kauffman.

Our KSI leadership has had a close long personal association with Dr. Joel D. Isaacson, who discovered the RD phenomenon in the 1960s and has been the lead scholar-scientist in its definition and research ever since. He stated that RD is a fundamental part of perception, cognition, and intelligence, and he suggested that this idea is "a law of nature, perhaps on a par with gravity" (e-mail to Bob Krone, April 20, 2011).

The important scientific paper co-authored by Dr. Isaacson and Dr. Louis H. Kauffman in this Special Science issue of the *Journal* captures the current understandings and implications of the RD intelligence reality, which may have existed from the Big Bang and which is hypothesized to be equally prevalent in the universe to Energy – Albert Einstein's legacy – and to Gravity – Sir Isaac Newton's legacy.

We consider this to be the most important scientific issue to date of the *Journal of Space Philosophy*.



Recursive Distinctioning: The Root of Nature's Cosmic Intelligence

By Bob Krone, Ph.D., Editor-in-Chief, Journal of Space Philosophy

I have been asked to describe for readers, in layperson terms, Recursive Distinctioning (RD), the scientific term for a fundamental natural process in the universe discovered by Dr. Joel D. Isaacson at Goddard Space Flight Center in 1964. Readers not familiar with astrophysics, cybernetics, or advanced mathematics must realize that attempts to describe complex space phenomena in non-scientific terms are bound to misrepresent their substance. For example, putting Einstein's 1915 Theory of General Relativity into the popular vernacular remains a dilemma even today.

"Nature's Cosmic Intelligence," the title of Dr. Isaacson's article in the Fall 2012 issue of the *Journal of Space Philosophy*,¹ is the best short description we have today. The origin of RD as a natural phenomenon is unknown. We can hypothesize that it began with the Big Bang, but there will never be proof of just when it began. It was not discovered until 1964, because there had been no way of detecting it. Dr. Isaacson patented the RD process in 1981 and noted that it had a striking link to the Hegelian dialectic (i.e., thesis—antithesis—synthesis), which is believed to underlie patterns of human thought processes. RD processes are also generators of patterns of elementary particles, called baryons, described through their quark constituents, and RD also seems to be a blueprint for DNA replication.

RD operates in what is called *streak mode* via recursion on units with boundaries that have distinctions from each other. RD is fundamental to human cognition and to other living things, including certain intelligent behaviors of bacteria. It may also be a universal mode of communication among diverse intelligent species in the universe. In the view of Dr. Louis H. Kauffman, Professor of Mathematics at the University of Illinois at Chicago, the properties of recursion and distinction underlie all of mathematics. RD combines distinction and recursion in a fundamental way, the consequences of which will be very important. RD work has evolved into a joint project between Joel D. Isaacson and Louis H. Kauffman. There is no demarcation line between their respective contributions in regards to RD *per se*.

There are strong indications that RD is a basis for many developments in many fields, including computing artifacts that mimic natural intelligence. The potential for significant impacts of RD across many sciences and technologies remains to be identified through research. The discovery that our universe contains information and intelligence in a process that is basic also to human perception and cognition (i.e., thinking) is a paradigm shift in scientific knowledge. Dr. Isaacson and Dr. Kauffman are making a huge contribution to Cosmos understanding. Readers should also consult Bob Krone's 2014 article, "Isaacson 1980 Aspirational Statement – Space Exploration."²

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¹ *Journal of Space Philosophy* 1, no. 1 (Fall 2012), 8-16.

² *Journal of Space Philosophy* 3, no. 1 (Spring 2014), 146-50.

Recursive Distinctioning

By Joel Isaacson and Louis H. Kauffman

Abstract

In this paper we explore Recursive Distinctioning.

Keywords: Recursive Distinctioning, algebra, topology, biology, replication, cellular automaton, quantum, DNA, container, extainer

1. Introduction to Recursive Distinctioning

Recursive Distinctioning (RD) is a name coined by Joel Isaacson in his original patent document¹ describing how fundamental patterns of process arise from the systematic application of operations of distinction and description upon themselves.² Louis H. Kauffman has written several background papers on recursion, knotlogic, and biologic.³

RD means just what it says. A pattern of distinctions is given in a space based on a graphical structure (such as a line of print, a planar lattice, or a given graph). Each node

¹ Joel D. Isaacson, "Autonomic String-Manipulation System," US Patent 4,286,330, August 25, 1981, www.iss.org/2001meet/2001paper/4286330.pdf.

² See also Joel D. Isaacson, "Steganographic Representation of the Baryon Octet in Cellular Automata," archived in the 45th ISSS Annual Meeting and Conference: International Society for the System Sciences, Proceedings, 2001, www.iss.org/2001meet/2001paper/stegano.pdf; Joel D. Isaacson, "The Intelligence Nexus in Space Exploration," in *Beyond Earth: The Future of Humans in Space*, ed. Bob Krone (Toronto: Apogee Books, 2006), Chapter 24, thespaceshow.files.wordpress.com/2012/02/beyond_earth-ch24-isaacson.pdf; Joel D. Isaacson, "Nature's Cosmic Intelligence," *Journal of Space Philosophy* 1, no. 1 (Fall 2012): 8-16.

³ Louis H. Kauffman, "Sign and Space," in *Religious Experience and Scientific Paradigms: Proceedings of the 1982 IASWR Conference* (Stony Brook, NY: Institute of Advanced Study of World Religions, 1985), 118-64; Louis H. Kauffman, "Self-reference and recursive forms," *Journal of Social and Biological Structures* 10 (1987): 53-72; Louis H. Kauffman, "Special Relativity and a Calculus of Distinctions," in Proceedings of the 9th Annual International Meeting of ANPA (Cambridge: APNA West, 1987), 290-311; Louis H. Kauffman, "Knot Automata," in *Proceedings of the 24th International Conference on Multiple Valued Logic – Boston* (Los Alamitos, CA: IEEE Computer Society Press, 1994), 328-33; Louis H. Kauffman, "Eigenform," *Kybernetes* 34, no. 1/2 (2005): 129-50; Louis H. Kauffman, "Reflexivity and Eigenform – The Shape of Process," *Constructivist Foundations* 4, no. 3, (July 2009): 121-37; Louis H. Kauffman, "The Russell Operator," *Constructivist Foundations* 7, no. 2 (March 2012): 112-15; Louis H. Kauffman, "Eigenforms, Discrete Processes and Quantum Processes," *Journal of Physics, Conference Series* 361 (2012): 012034; Marius Buliga and Louis H. Kauffman, "Chemlambda, Universality and Self-Multiplication," in *Artificial Life 14 – Proceedings of the Fourteenth International Conference on the Synthesis and Simulation of Living Systems*, ed. Hiroki Sayama, John Rieffel, Sebastian Risi, René Doursat, and Hod Lipson (Cambridge, MA: MIT Press, 2014); Louis H. Kauffman, "Iterants, Fermions, and Majorana Operators," in *Unified Field Mechanics – Natural Science Beyond the Veil of Spacetime*, ed. R. Amoroso, L. H. Kauffman, and P. Rowlands (Singapore: World Scientific, 2015), 1-32; Louis H. Kauffman, "Biologic," *AMS Contemporary Mathematics Series* 304 (2002): 313-40; Louis H. Kauffman, "Self-Reference, Biologic and the Structure of Reproduction," *Progress in Biophysics and Molecular Biology* 119, no. 3 (2015): 382-409; Louis H. Kauffman, "Biologic II," in *Woods Hole Mathematics*, ed. Nils Tongring and R. C. Penner, World Scientific Series on Knots and Everything, Vol. 34 (Singapore: World Scientific, 2004), 94-132; Louis H. Kauffman, "Knot Logic," in *Knots and Applications* (Singapore: World Scientific, 1994), 1-110; Louis H. Kauffman, *Knots and Physics*, 4th ed. (Singapore: World Scientific, 2012).

of the graph is occupied by a letter from some arbitrary alphabet. A specialized alphabet is given that can indicate distinctions about neighbors of a given node. The neighbors of a node are all nodes that are connected to the given node by edges in the graph. The letters in the specialized alphabet (call it SA) are used to describe the states of the letters in the given graph and at each stage in the recursion, letters in the SA are written at all nodes in the graph, describing its previous state. The recursive structure that results from the iteration of descriptions is called RD. Here is an example: we use a line graph and represent it just as a finite row of letters. The alphabet is $SA = \{=, [,], O\}$ where “=” means that the letters to the left and to the right are equal to the letter in the middle. Thus if we had AAA in the line then the middle A would be replaced by =. The symbol “[” means that the letter to the *left* is different. Thus in ABB the middle letter would be replaced by [. The symbol “]” means that the letter to the right is different. And finally the symbol “O” means that the letters both to the left and to the right are different. SA is a tiny language of elementary letter distinctions. Here is an example of this RD in operation where we use the proverbial three dots to indicate a long string of letters in the same pattern. For example,

AAAAAAAAABAAAAAAAAA

is replaced by

=====]O[=====

is replaced by

=====]OOO[=====

is replaced by

=====]O[=]O[===== .

Note that the element]O[appears and that it has replicated itself in a kind of mitosis. See Figures 1 and 2 for a more detailed example of this evolution. In Figure 3 we show the evolution of the RD, starting from a more arbitrary string. Elementary RD patterns are fundamental and will be found in many structures at all levels. To see a cellular automaton example of this phenomenon of patterns crossing levels of structure, we later look at a replicator in “HighLife” a modification of John Horton Conway’s automaton “Life.” The HighLife replicator follows the same pattern as our RD replicator. However, the entity in HighLife that is self-replicating requires twelve steps to do the replication. The resultant patterns of replication can be seen in Figures 54 to 61. In the successive figures, twelve steps are hidden and we see the same basic pattern shown in Figure 1. We can understand directly how the RD replicator works. This gives a foundation for understanding how the more complex HighLife replicator behaves in its context. We take this phenomenon of the simple and the complex to be generic for many systems. By finding a point of simplicity, we make possible the evolution of understandings that are otherwise impossible to obtain.

```
*AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA*
*          ]o[          *
*          ]ooo[        *
*          ]o[ ]o[      *
*          ]oooooooo[    *
*          ]o[   ]o[     *
*          ]ooo[ ]ooo[   *
*          ]o[ ]o[ ]o[ ]o[ *
*          ]ooooooooooooo[ *
*          ]o[           ]o[ *
*          ]ooo[         ]ooo[ *
*          ]o[ ]o[       ]o[ ]o[ *
*          ]oooooooo[     ]oooooooo[ *
*          ]o[   ]o[   ]o[   ]o[ *
*          ]ooo[ ]ooo[ ]ooo[ ]ooo[ *
*          ]o[ ]o[ ]o[ ]o[ ]o[ ]o[ ]o[ ]o[ *
*          ]oooooooooooooooooooooooooooooooooooo[ *
*          ]o[                   ]o[ *
*          ]ooo[                   ]ooo[ *
*          ]o[ ]o[                   ]o[ ]o[ *
*          ]oooooooo[                   ]oooooooo[ *
*          ]o[   ]o[                   ]o[   ]o[ *
*          ]ooo[ ]ooo[                   ]ooo[ ]ooo[ *
*          ]ooo[ ]ooo[                   ]ooo[ ]ooo[ *
*          ]ooo[ ]ooo[                   ]ooo[ ]ooo[ *
```

Figure 1: RD replication

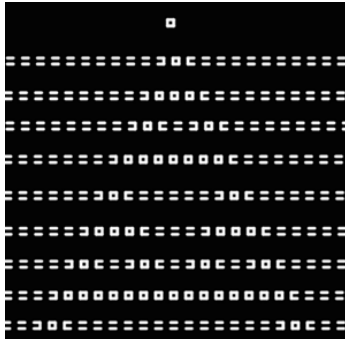


Figure 2: Second picture of RD replication

```

*GEEEEENNONSESTREEENG*
*=O[==][ ]OOOOOOOOO[=]O=*
*=OO[=]OOO[=====]OOO=*
*=[]OOO[=]O[=====]O[==]*
*=OO[=]OOOOO[====]OOO[ ]O=*
*=[]OOO[==]O[==]O[=]OOO=*
*=OO[=]O[=]OOO[ ]OOOOO[=]*
*=[]OOOOOOO[=]OO[==]OOO=*
*=OO[=====]OOO[ ]O[=]O[=]*
*=[]O[==]O[=]OOOOOOOOO=*
*=OOOO[=]OOOOO[=====]=*
*=[==]OOO[==]O[=====]O=*
*=O[ ]O[=]O[=]OOO[====]OO=*
*=OOOOOOOOOOO[=]O[==]O[ ]=*
*=[=====]OOOOO[ ]OOO=*
*=O[=====]O[==]OO[==]*
*=OO[=====]OOO[=]O[ ]O[ ]O=*
*=[]O[==]O[=]OOOOOOOOO=*
*=OOOO[=]OOOOO[=====]=*
*=[==]OOO[==]O[=====]O=*
*=O[ ]O[=]O[=]OOO[====]OO=*
*=OOOOOOOOOOO[=]O[==]O[ ]=*
*=[=====]OOOOO[ ]OOO=*

```

Figure 3: A string evolution

We can place the basic idea of RD with the context of cellular automata. RD is distinct from other types of cellular automaton in that its basic recursion is based on direct distinctions made (locally) in relation to distinctions present in the given state of the automaton. In a typical cellular automaton, the next state is obtained on the basis of simple distinctions about the previous state. These distinctions are not necessarily at the letter level. For example, in a Wolfram line automaton we have eight possible local neighborhoods consisting of triples of zeros and ones.

Any distinction made among these eight, separating them into two classes, is acceptable as a rule for the Wolfram automaton. The operation of distinction is shifted to a higher level than the question of sameness or difference for nearby iconic elements of the state. This is the distinction between our “orthodox” RD models and other recursive models. We are interested in rules that involve direct matters of sameness or difference. Such RD rules are very primitive rules. Nevertheless, we regard the orthodox RD models as part of the larger class of recursive cellular automata. We wish to explore the relationships between our primordial structures and the closely related structures of all cellular automata as they are understood at this time.

Everyone who works in science, mathematics, or computer science is familiar with the fundamental role of the concept of distinction and the making of distinctions in both theory and practice. For example, Einstein's relativity depends on a new distinction between space and time relative to an observer and a new unification of space and time that is part and parcel of this distinction. Every moment of using a digital computer depends upon the myriad of distinctions that the computer handles automatically, enabling the production and recording of these words and the computation and transmission of information. Distinctions act on other distinctions. Once a new distinction is born, it becomes the object of further action. Thus grows all the physics that comes from relativity and thus grows all the industry of computation that grows from the idea and implementation of the Turing machine, the programmed computer.

And yet it is not usually recognized that it is through RD that all such progress is made. We discuss RD both in its human and its automatic aspects. In the automatic aspect, we give examples of automata that are based on making very simple distinctions of equality and right/left that then, upon allowing these distinctions to act on themselves, produce periodic and dialectical patterns that suggest what are usually regarded as higher level phenomena. In this way, and with these examples, we can illustrate and speculate on the nature of intelligence, evolution, and many themes of fundamental science.

The remarkable feature of these examples of RD is their great simplicity coupled with the complexity of behaviors that can arise from them. Notice that each successive string in the recursion can be regarded as describing its predecessor. It is remarkable that there should be such intricate structure in the process of description. Description is another word for making a distinction. The description of a given string is a string of individual distinctions that have been made. Each individual distinction is one that recognizes whether a given character in a string is equal to a left neighbor, a right neighbor, both, or neither. This elementary distinction becomes instantiated as a character in the new description string. The description string can be subjected to the same scrutiny, and so the recursive process continues.

Note that this recursive process depends, at its base, on the most elementary distinctions possible for character strings. No mathematical calculations are performed. We should mention that distinction-making without mathematical computation is ubiquitous in natural neuronal processing. Joel Isaacson's collaboration with Eshel Ben-Jacob has included attempts to demonstrate RD in live neuronal tissue.⁴ One can also point to the molecular interactions of DNA and RNA as natural RD automata. Finally, we can point to Buliga and Kauffman's⁵ notion of chemlambda computation as abstract chemical combination computing that includes aspects of lambda calculus, but is based on direct and local action related to distinctions inherent in the system.

The epistemology behind this automaton is based directly on distinctions that can be made automatic. Other cellular automata are also based on distinctions. For example,

⁴ Private Communication with Eshel Ben-Jacob.

⁵ Buliga and Kauffman, "Chemlambda."

the well-known Wolfram line automata⁶ are based on character strings with only two characters and the recognition of the eight possible triples of characters, including characters to the left and to the right of a given character. The automaton rule then replaces the middle character according to the structure of this neighborhood. There is a crucial difference in epistemology between a Wolfram line automaton and our RD program. We do not replace according to an arbitrary rule. We place a character that describes the distinctive structure of the neighborhood of the predecessor character. Our automaton engages in a meta-dialogue about its own structure. This dialogue is then entered as a string for the automaton to examine and act upon once again. The patterns produced by this recursive distinction are part of a dialogue that the strings hold with themselves. One can ask many questions about RD as presented here. The automaton we have demonstrated illustrates a concept that can be instantiated in many ways. We hope, in a paper to come, to demonstrate Turing universality for automata of this type. But in fact we feel that the paradigm of RD goes beyond (or around) the paradigm of the Turing machine, and we will discuss that issue as well.

There is another level to our automaton and that is the level of examining with human eyes and minds the output of the automaton, seeing patterns in the whole collection of strings and engaging in further design on this basis. This is where the recursive automatic distinctions meet the aware distinguishing of the observers of the system, connecting the automatic with the aware process and design level that goes on in the larger network of science.

It is the case that in the design of computing machines human beings have for centuries confronted the issue of repeatability for the sake of computation or for the sake of the production of pattern (as in weaving) or the reliability of manufacture (as in timekeeping). This means that elementary distinctions must be reproducible and comparable as in mathematical notations, written language, and the mechanics of clocks and computing devices. Thus we shall refer to automatic distinctions when we speak of highly repeatable physical situations that can be regarded as reproductions of distinctions that are available to an observer. In some cases, such distinctions are designed by someone who engineers them into the device. In other cases, we recognize computational and reproducible patterns in natural situations. The earth goes around the sun periodically; the moon goes around the earth. Natural clocks arise from these periodicities and regularities observed in our world. Thus, in this essay, we do not restrict ourselves in the use of the word distinction to the meaning that a distinction is made by some human observer. We refer to distinctions that are ongoing in a device beyond our direct observation. Nevertheless, the buck stops at a human observer who recognizes the patterns of the device and who interprets the meaning of what has been produced. It is then possible to discuss the role of creativity in relation to deterministic and automatic actions.⁷

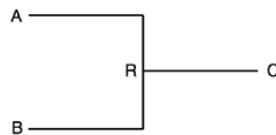
⁶ Stephen Wolfram, *A New Kind of Science* (Champaign, IL: Wolfram Media, 2012).

⁷ Acknowledgement. We thank Bernd Schmeikal for conversations and for sharing his own research in relation to our work. We thank Dan Sandin for a continuing collaboration with Lou Kauffman and particularly for sharing the computer program for 2D RD that has been evolved by the two of them. The graphical illustrations of 2D RD in this paper were all produced by that program. It also gives us great

2. The Logic of Distinction and the Distinction of Logic

We have introduced the one-dimensional RD and its very simple alphabet based on the four iconic symbols shown in Figure 4. In this figure, we use a box rather than a circle for the icon that indicates difference to both the right and the left, and we use a box with a missing left vertical edge to denote sameness on the left and a box with a missing right vertical edge to denote sameness on the right. Sameness on both right and left is indicated by the two parallel lines that remain when the two vertical edges of the box are removed. In this figure, we give a logical justification for these icons in terms of the act of discrimination. That is, we give a logical construction for an icon that describes and embodies the discrimination itself. At a given point in the line of letters, there is a given letter. This letter is either distinct or different from its neighbor to the left and/or its neighbor to the right. We introduce a method to *manufacture* an icon that expresses these distinctions. In order to do this, we insert a line segment in between the space for the given letter and the space next to it *if there is a difference between the given letter and its neighbor*. We take as given a line segment at the top of the space and a line segment at the bottom of the space. (This actually indicates the condition of the one-dimensional RD where it is distinct from its context above and below the one dimension of operations.) As a result, this process of discrimination constructs four possible icons that describe the condition of a given letter. The icons are illustrated (Figure 4), and the reader can see that they are an equals sign when there is no distinction to the left or to the right, left and right brackets when there is a distinction to the left or the right but not both, and a rectangular box when there is a distinction to both the right and the left. In the next few paragraphs, we describe this process further in terms of logical operations.

pleasure to acknowledge Tom Mandel. Fifteen years ago, on his own initiative, he posted Joel Isaacson's patent and his Stegano paper on the ISSS website, when he managed that site. Furthermore, we feel that the basic RD process is a clapping machine realizing part of Tom's vision for the notion of depicting a relationship as a picture where when the This and the That are the two hands, then the Clapping of the hands connotes the relationship that is brought forth. In the RD, it is the distinctions and the spaces between them that clap in time and produce the "sounds" of further distinctions. Tom uses the notation shown below,



or algebraically, $(A,B)R = C$, where C stands for the (whole) dividing/arising from A and B, and R the connection/relation of A and B; the This and the That. Such notation is simple, yet insistent, calling for the articulation of the unity C, the relation R and the "parts" A and B: Why is this important? The answer is: Because such notation and the attitude behind it continually call the question of relationship and the nature of relationship. All descriptions, all systems, are built this way. But we keep forgetting the glue and putting it into the background. Here, all three fundamentals in any distinction are brought into the foreground.

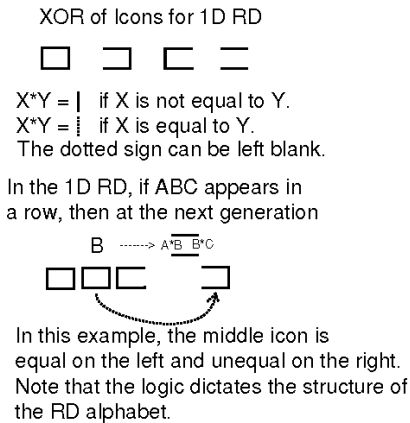


Figure 4: XOR of icons

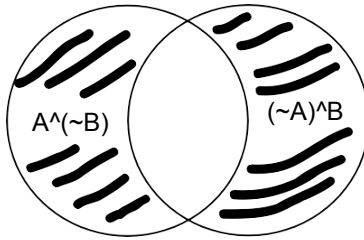
The fundamental underlying operation is exclusive or, often denoted by XOR. When we say “A XOR B” we mean the statement “A or B but not both A and B.” This special version of OR has the property that it is true only when A and B have different truth values. Logically, “A or B but not both A and B” is equivalent to “A and not B, or B and not A.” In this form we write the formula

$$A * B = (A \wedge (\sim B)) \vee ((\sim A) \wedge B).$$

Here $A * B$ denotes “A XOR B”, $A \vee B$ denotes “A or B,” and $A \wedge B$ denotes “A and B.”

When working with sets, we can interpret $A * B$ as the intersection of A with the complement of B taken in union with the intersection of the complement of A with B. This is illustrated in Figure 5. In using the Venn diagrams, we have a very intuitive interpretation of XOR. A set is denoted by a shaded circle and when we XOR two sets, the part where they overlap vanishes. Thus two identical sets will yield an empty diagram under this operation. In this sense, a set is its own negation! We return to this point of view in Section 10 when we discuss the relationship of RD with Spencer-Brown’s Laws of Form. In letting one shaded region operate upon another, the parts that remain black after the XOR operation indicate the differences between the two sets. In this way, XOR is a logical exemplar of the operation of discrimination and it can be understood to underlie all the RD operations we describe. One can imagine that discrimination (as practiced by thinking beings) is more complex than XOR, but XOR is a backbone or skeletal aspect of all instances of discrimination.

A or B but not both A and B.



$$A \text{ XOR } B = A * B = A^{(\sim B)} \vee (\sim A)^B$$

Figure 5: XOR in Venn diagrams

Now view Figure 4 once more. Here we show explicitly how the XOR operation acts on the icons for the 1D RD to produce the icons at the next iteration. We use a vertical slash | and an unmarked vertical slash / for the two states of discrimination. We call these the *marked* and *unmarked* states, respectively. Given two such states, we define $A * B$ as marked if one of A and B are different. If A equals B, then $A * B$ is unmarked. This construction is then applied to the local interactions of the icons in the RD. If we have a row with ABC in that row, then for the new B, we form $A * B$ and $B * C$. These vertical slashes or unmarked slashes become the left and right ends of the new icon that represents the new B in the next row of the RD, one full time-step later. Thus the new icon is formed by the discriminations to its left and to its right in regard to those neighbor icons. The figure shows explicitly how we leave the horizontal lines of the icon unchanged while we change the vertical slashes. As mentioned at the beginning of this section, this means that the logic of left and right naturally creates the four icons that are used in the 1D RD. The alphabet arises in the act of discrimination. The act of discrimination is quite general for the RD. Any letters or icons can be given to it at the start. The XOR applies to make the discrimination and to produce a standard icon that indicates the left-right discrimination that was made.

Now view Figure 6, where we indicate how the XOR process can be accomplished by digital circuitry. The figure should be self-explanatory. There is a basic inverting element that will take states to their opposites and, with a multiplicity of inputs, this inversion is regarded as a NOR gate. That is, one starts with a collection of variables $\{a, b, c, d\}$ and the NOR gate returns $\sim (a \vee b \vee c \vee d)$. The circuit then implements the formula for the XOR operation that we have given above. This means that we could have an RD automaton that sampled signals inside a larger digital environment. It also means that we can look at the RD as connected inside an information-processing environment that uses logical operations in great generality. In particular, one could think of a sensing device that can detect differences in signals with which it otherwise has no direct access. Isaacson⁸ has called such external but not directly detectable signals *fantomarks*. The information about their differences can become the initial data for an RD system that then amplifies and modifies these patterns, allowing the possibility for communication (by letting another system find differences in the signals generated by

⁸ Isaacson, "Autonomic String-Manipulation System."

this RD) between systems that have internal states that are fantomarked for the other system. Isaacson has speculated that this could be the basis for communicating with extraterrestrials. Here we point out that it can be regarded as a partial description of the situation of human-to-human communication with its mix of local-to-global discrimination based on the detection and articulation of differences.

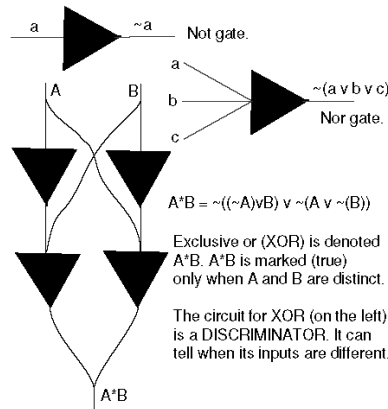


Figure 6: XOR circuit

We regard this description of the process of discrimination to be fundamental. A ground that is subject to discrimination is given at the beginning. The XOR operations probe this ground and write naturally via marked and unmarked states in the geometry and alphabet of special icons that can be further discriminated by the same process. The icons record a neighborhood of discriminations. In the case of 1D RD, this neighborhood is described in terms of left and right. The process of discrimination alternates between the local indications of marked and unmarked states (the vertical slash and its absence) and the global examination of icons for their identity or difference. It is this crossing of levels that makes the structure of the RD process repeatable and unique.

In general, an RD structure has alphabetic elements at specific loci. A process of discrimination generates an icon for that location that describes the distinctions between that letter and its neighbors. These icons of distinction become the letters of a special alphabet that is coherent with the geometry of the RD structure. The recursion replacing present icons or alphabetic elements with these icons of distinction is the process of RD. The process arises directly from the idea of description and the fundamental distinction of the given geometry. In the next section, we show how this works for two-dimensional RD.

3. Two-Dimensional RD, a 16-Letter Alphabet, Quaternions and Spacetime

We now consider a natural generalization of the one-dimensional RD to two dimensions. The geometry of the 2D RD is a rectangular lattice with square cells. Each cell is regarded as having four neighbors, one to the north, one to the south, one to the east, and one to the west, each sharing a one-dimensional interval of common

boundary. The simplest occupant of such a cell corresponds to openings or closings of the four parts of the boundary. Thus one can block all of the boundary, or all but one edge of the boundary, or all but two edges of the boundary and continue until one has the unique empty icon with no edges from the boundary. This makes a 16-letter alphabet, as illustrated in Figures 7 and 8.



Figure 7: A snapshot of a 2D RD

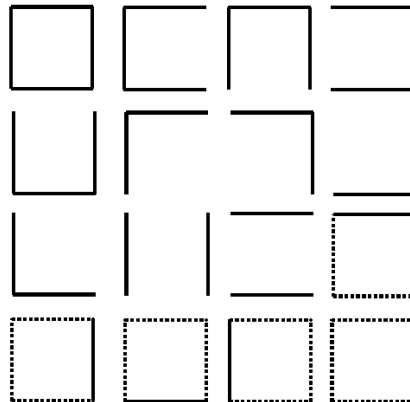


Figure 8: The 2D alphabet 1

In Figures 9 and 10, we indicate how to code the letters as ordered sequences of four elements, each element a plus or a minus sign. In these figures, we also indicate how to make XOR combinations of these edges of the icons. The rule is simply that the superposition of two edges cancels them. With this, we can combine the letters to form other letters by superimposing them. When two letters are identical, then the superposition is the empty letter. Otherwise it is not empty, and it is a new resultant

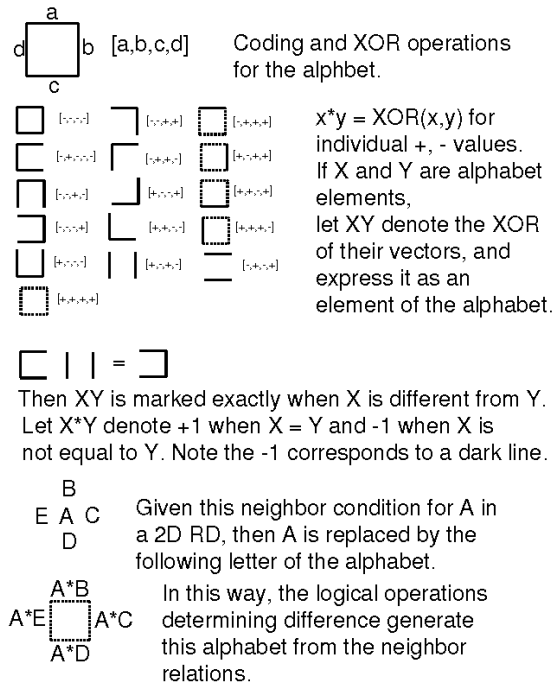


Figure 10: The 2D alphabet 3

In the sequence from Figure 11 to Figure 19, we show eight steps from the first figure and returning to that figure. The first figure is an empty box with a fixed boundary condition that declares that its outer squares are different from the adjacent squares outside the box. Each successive figure is the result of one redescription by the RD process. In this case and with this initial condition, the process has period eight.

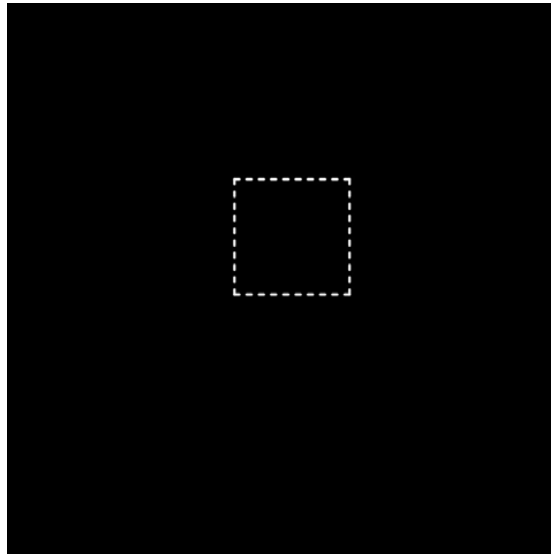


Figure 11: 2D RD box, no seed

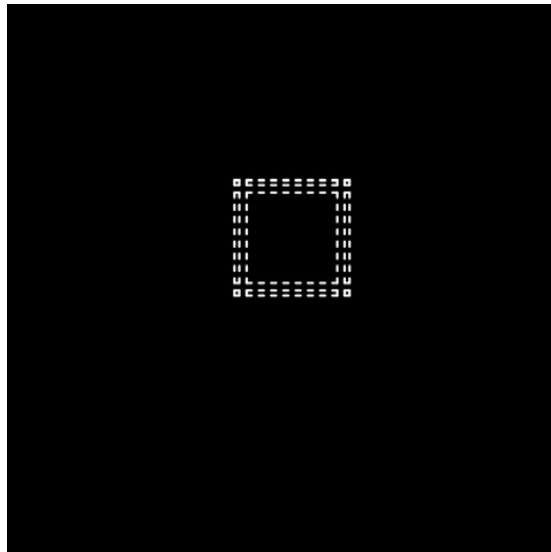


Figure 12: 2D RD box, no seed

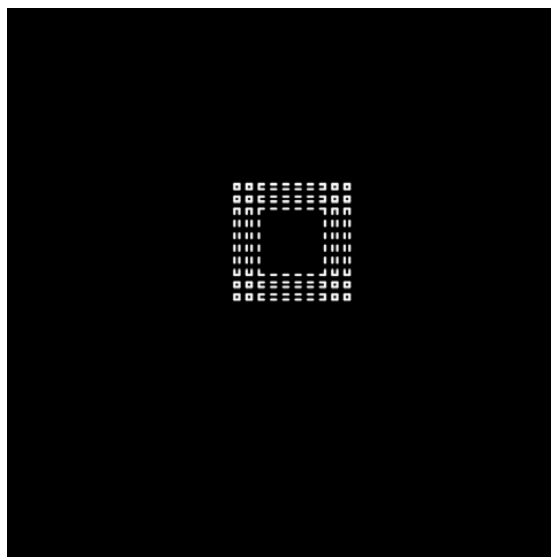


Figure 13: 2D RD box, no seed

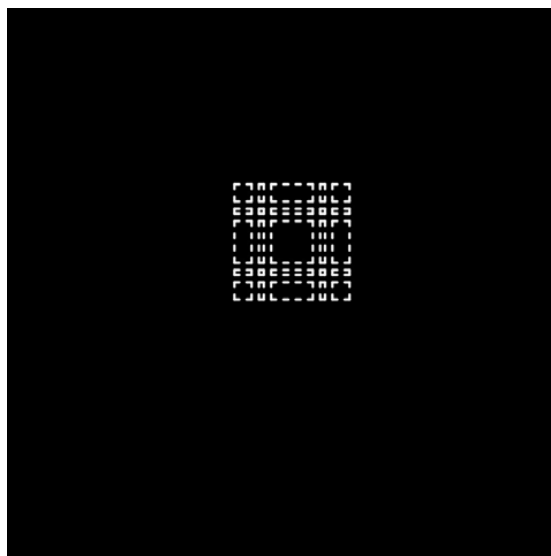


Figure 14: 2D RD box, no seed

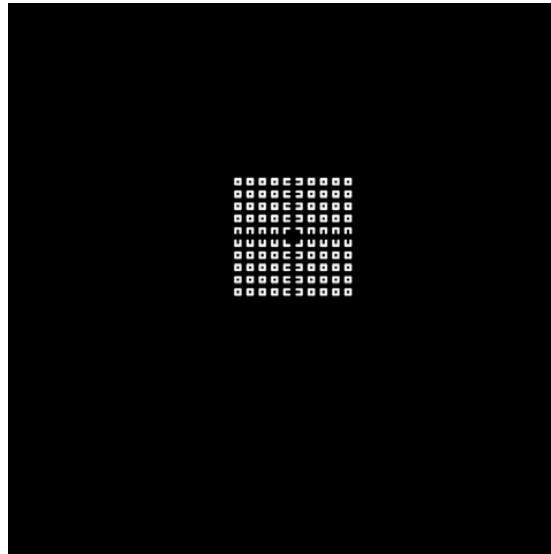


Figure 15: 2D RD box, no seed

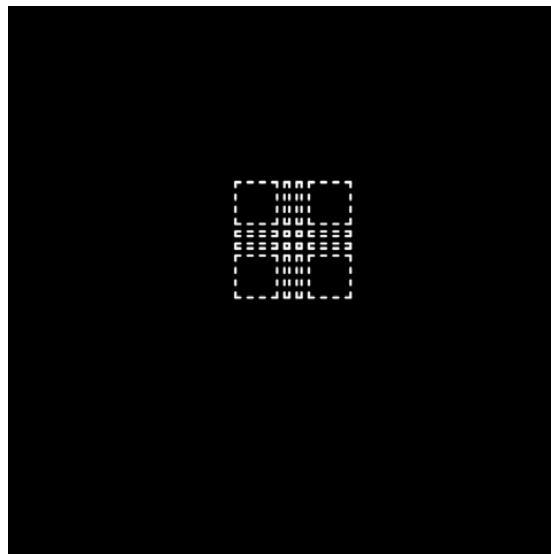


Figure 16: 2D RD box, no seed

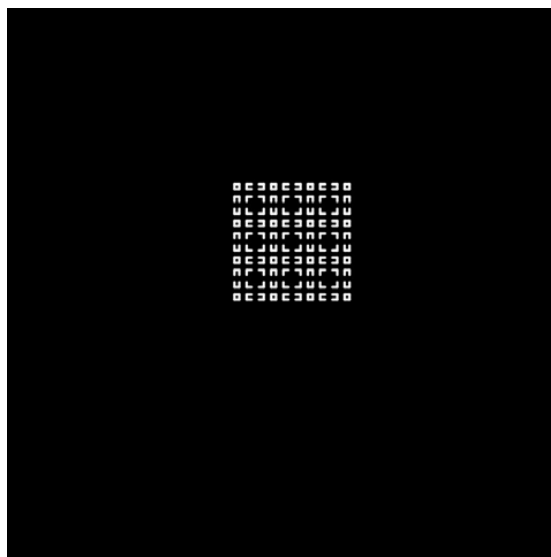


Figure 17: 2D RD box, no seed

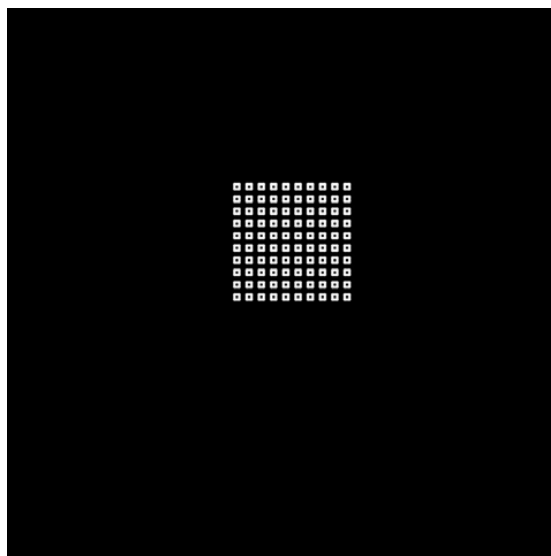


Figure 18: 2D RD box, no seed

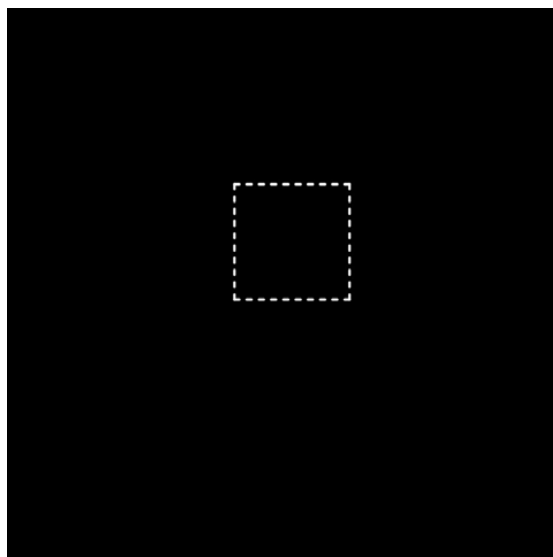


Figure 19: 2D RD box, no seed

In the sequence from Figure 20 to Figure 32, we show the same box with a different initial condition (some marked spaces inside). Now the evolution is more complex, as is illustrated in the figures. Remarkably, in this case the result is eventually periodic of period two.

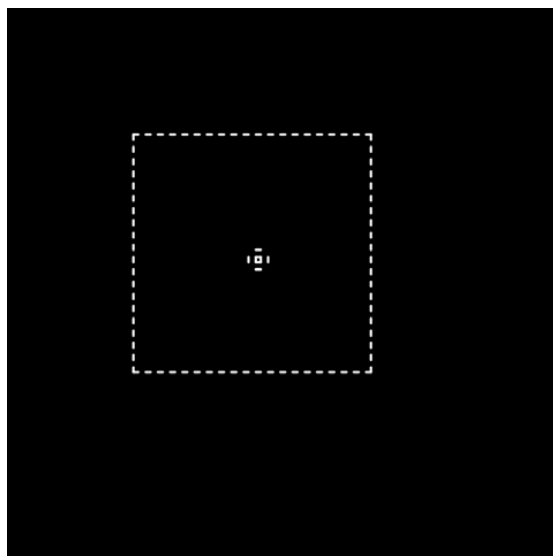


Figure 20: 2D RD box with seed

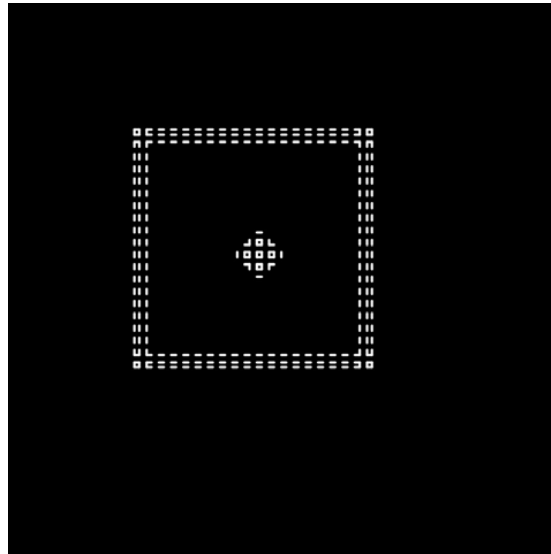


Figure 21: 2D RD box with seed

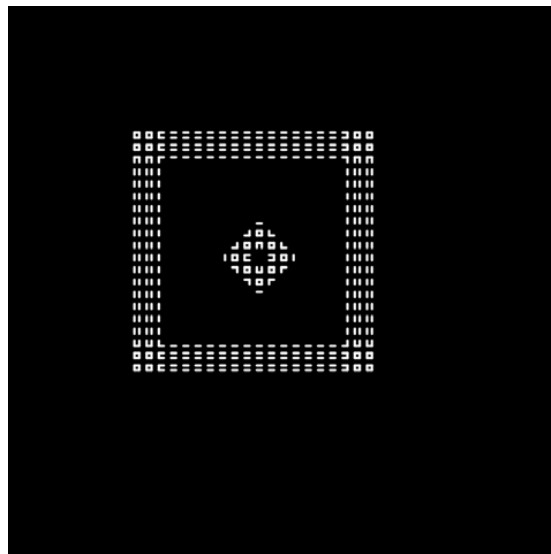


Figure 22: 2D RD box with seed

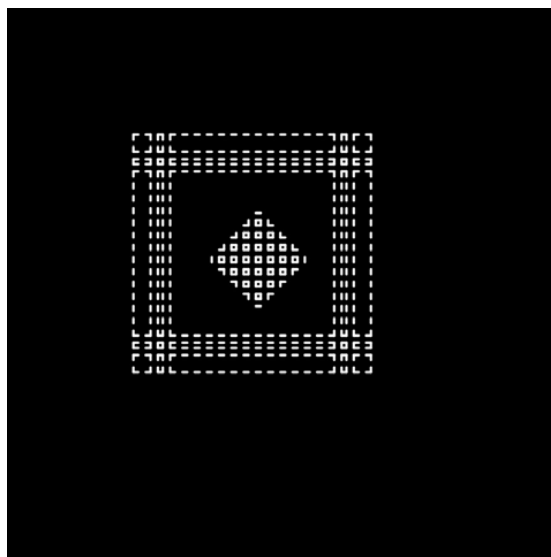


Figure 23: 2D RD box with seed

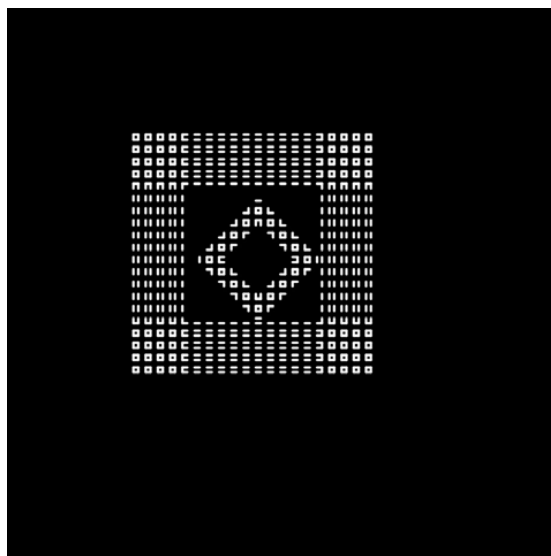


Figure 24: 2D RD box with seed

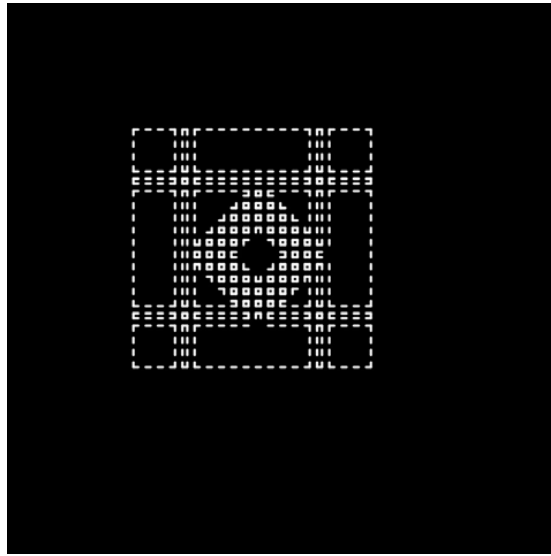


Figure 25: 2D RD box with seed

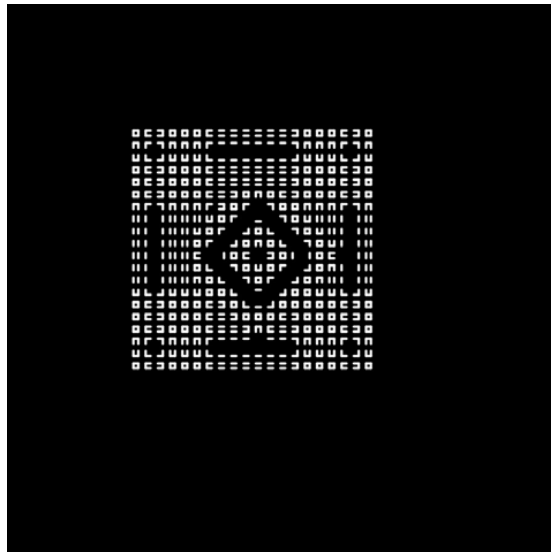


Figure 26: 2D RD box with seed

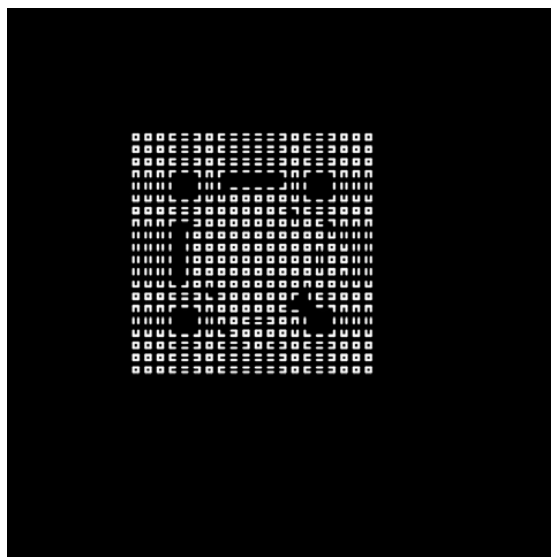


Figure 27: 2D RD box with seed

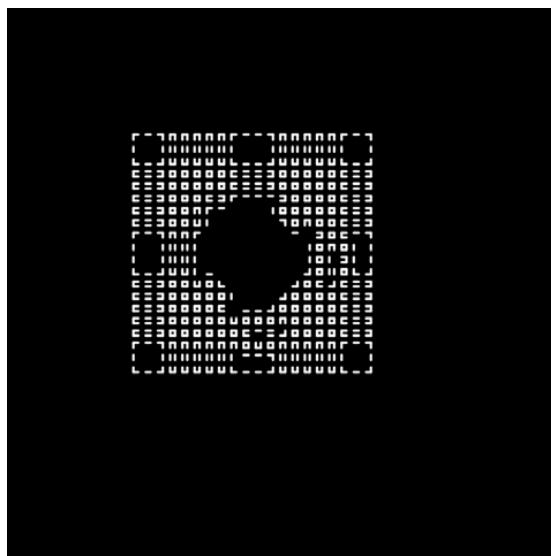


Figure 28: 2D RD box with seed

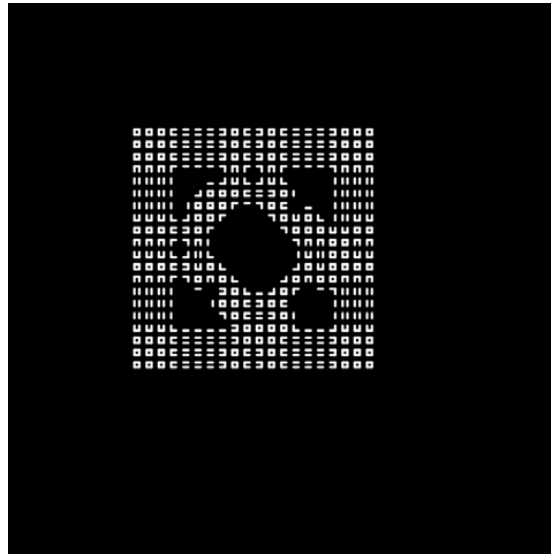


Figure 29: 2D RD box with seed

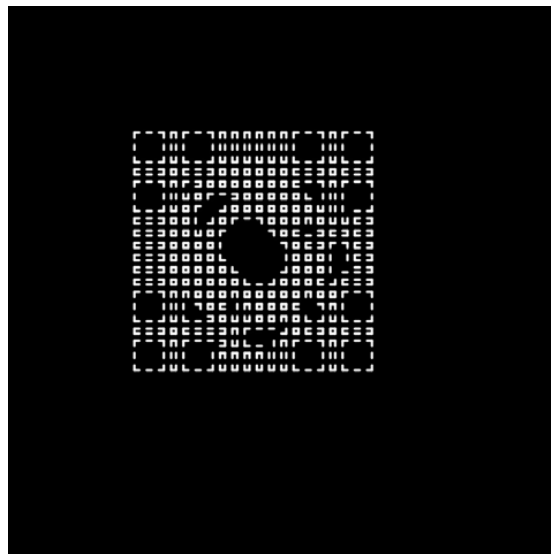


Figure 30: 2D RD box with seed

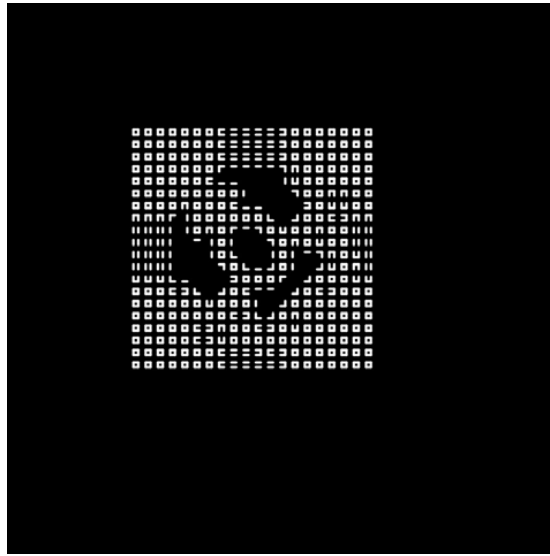


Figure 31: 2D RD box with seed

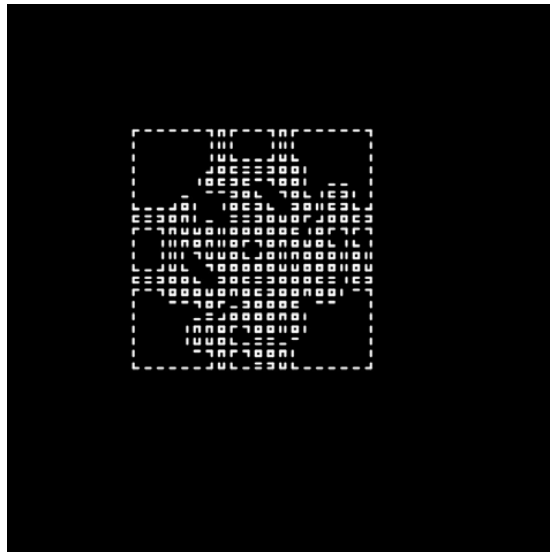


Figure 32: 2D RD box with seed

Figure 33 and Figure 34 illustrate two consecutive frames from this automaton after it has entered period two. The reader can compare these two frames and see that each describes the other. Focus on the pair of 2D patterns, Tweedledum and Tweedledee, in these two figures. What is remarkable about these two patterns is that they mutually describe each other in such a way that they complement each other, just like a positive and a negative in photography. If separated, each would construct its complement, and the patterns would replicate indefinitely. So these are antithetical and their superposition yields a synthesis. (A synthesis here would be the big square filled completely with only little squares.) Note that they are typical in many 2D RD runs and are not exceptions.

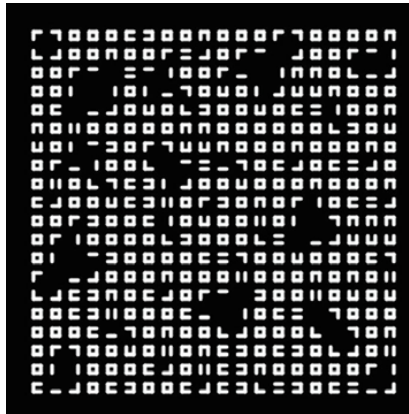


Figure 33: Tweedledum

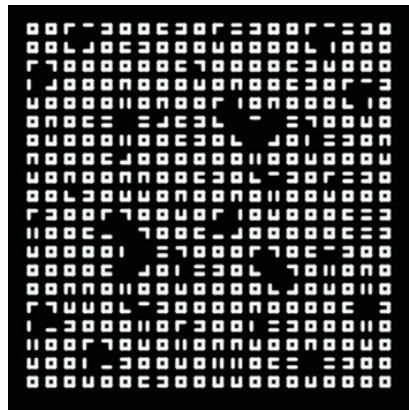


Figure 34: Tweedledee

The two strands of DNA are also complementary, which allows their replication. The reader will recognize how much more complex this 2D complementarity is than the 1D complementarity of DNA. Obviously, no one can dream of or design such intricate mutual descriptions of patterns, and yet they are by-products of an automatic RD automaton. One might speculate that the DNA molecule with its complementary Watson and Crick strands evolved through recursive chemical interactions.

3.1 Quaternions and Iterants

In this subsection, we show how the 16-letter alphabet is related to the algebra of the quaternions and concomitantly to the algebra of spacetime. Before we do this, however, it will be helpful to explain a way to think about such matters that is developed in the paper by Kauffman (and the references therein).⁹ In that paper, one finds a temporal interpretation of the square root of minus one. The idea is that one starts with a simple oscillation such as

$$+ - + - + - + - \dots$$

⁹ Kauffman, "Iterants, Fermions, and Majorana Operators."

Starting in this way, we can connect with RD simply by observing that some of the simplest 1D RD with tight boundary conditions will oscillate with period two. Once recursion is on the scene, the simplest oscillations are inevitably present. That said, let us make two abbreviations that correspond to two ways to distinguish a period two oscillation:

$$[+, -] = [+1, -1]$$

and

$$[-, +] = [-1, +1].$$

These two ordered pairs correspond to distinguishing the oscillation as proceeding from plus to minus or as proceeding from minus to plus.

Call an ordered pair such as $[a, b]$ an *iterant*. We can combine iterants by adding their coordinates or by multiplying their coordinates.

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b][c, d] = [ac, bd].$$

We add to this structure an operator η that participates in the *time shift* that relates one iterant to the other.

$$\eta^2 = 1$$

$$[a, b]\eta = \eta[b, a].$$

Formally, η acts as a permutation of order two, exchanging $[a, b]$ for $[b, a]$ when it is commuted with an iterant. We regard an element of the form $[a, b]\eta$ as a *temporally sensitive iterant*. Note what happens when we multiply

$$i = [+1, -1]\eta$$

by itself.

$$i^2 = ii = [+1, -1]\eta[+1, -1]\eta = [+1, -1][-1, +1]\eta\eta = [(+1)(-1), (-1)(+1)]1 = [-1, -1] = -1.$$

Thus

$$i^2 = -1.$$

We have produced a square root of minus one as a temporally sensitive iterant associated with an elementary oscillation.

In fact, we have produced an algebra containing

$$\{\eta, 1 = [1, 1], -1 = [-1, -1], \alpha = [1, -1], -\alpha = [-1, 1]\}.$$

Note that

$$\eta^2 = \alpha^2 = 1$$

and that

$$\alpha\eta + \eta\alpha = 0.$$

This is a first example of a *Clifford algebra*, an algebra generated by elements of square one that anti-commute with one another. We have $i = \alpha\eta$ and

$$i^2 = \alpha\eta\alpha\eta = \alpha(-\alpha)\eta^2 = -\alpha^2 = -1.$$

Thus, we can also see our temporal interpretation of the square root of minus one as a Clifford algebra phenomenon.

Clifford algebras are deeply connected with physics. To see a hint of this we consider a fundamental formula from special relativity theory (we use the convention that the speed of light is $c = 1$). Let E denote energy, p momentum, and m the mass of a particle. Now let

$$E = \alpha p + \eta m.$$

Assume that p and m commute with α and β . You can easily prove by multiplying it out that

$$E^2 = (\alpha p + \eta m)(\alpha p + \eta m) = \alpha^2 p^2 + \eta^2 m^2 + (\alpha\eta + \eta\alpha)pm = p^2 + m^2 + 0pm = p^2 + m^2.$$

This formula $E^2 = p^2 + m^2$ is fundamental to special relativity, and we have shown that it follows from a Clifford algebra representation of the energy. This way of writing the energy is due to the great physicist Dirac, and is the beginning of the deep relationship between Clifford algebra and physics. Our point is that by looking at this through the lens of iterants, we can draw a connection between fundamental recursion and quantum and relativistic physics.¹⁰

Now we turn to the *quaternions*. Sir William Rowan Hamilton discovered quaternion algebra in 1843, after 15 years of trying to find a three-dimensional analog for complex numbers. When he realized the key was a four-dimensional space, the pattern fell into place. Recall that the quaternions are generated by $\{1, -1, I, J, K\}$ so that $I^2 = J^2 = K^2 = IJK = -1$ from which it follows that $IJ = K$, $JK = I$, and $KI = J$, and that $IJ = -JI$, $JK = -KJ$, and $KI = -IK$.

There is a natural iterant structure for the quaternions (see Figure 35). In this figure we show the order four iterant sequences that correspond to each of I , J , and K and the analogy of the simple time shifter η that is associated with each one. These analogs are

¹⁰ For further details, see Bernd Schmeikal, "Basic Intelligence Processing Space," *Journal of Space Philosophy* 5, no. 1 (Spring 2016): 65-89; Kauffman, "Iterants, Fermions, and Majorana Operators."

diagrammed as permutations, and they act when one composes the iterants by attaching their braided forms together. The new temporal shift operators generate the so-called Klein Four Group, the symmetries of a square.¹¹ We now show how this iterant version of the quaternions is related to our 16-letter alphabet and how the symmetries of the square come into play directly.

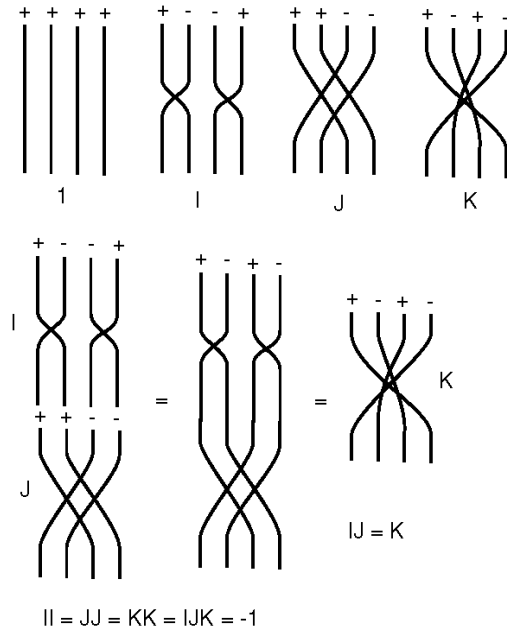


Figure 35: Iterant representation of the quaternions

Now we turn to Figure 36, where we show how there is a natural quaternion structure associated with the 16-letter alphabet. What you see is a subset of the 16-letter alphabet and the operations A, B, C (and 1) of the Klein Four Group. We define I, J, K each of the form $I = aA$, $J = bB$, and $K = cC$ where a, b, and c are certain elements of the 16-letter alphabet. We then define, e.g., $xA = Ax^A$, where x^A is the operation of the symmetry element A on the letter x. We define xy (on letters) via XOR of the corresponding letters in the alphabet. We find that I, J, and K give the quaternions. Thus the quaternions are a combination of XOR operations and symmetry operations in the alphabet. Note that $xy = \text{XOR}(x,y)$ = the result of superimposing x and y as letters and canceling common occurrences. Once we have the quaternions, we have an entry into spacetime algebra as follows. We have $II = JJ = KK = IJK = -1$. Let $E = (x,y,z,t) = xI + yJ + zK + t1$ where x, y, z, and t are real numbers. Then think of E as a point in spacetime. We have

$$E^2 == (xI + yJ + zK + t1)(xI + yJ + zK + t1) = -x^2 -y^2 -z^2 +t^2$$

¹¹ See Kauffman, "Iterants, Fermions, and Majorana Operators" for more details.

Which is the Minkowski metric (it is often written as the negative of this expression) for spacetime.

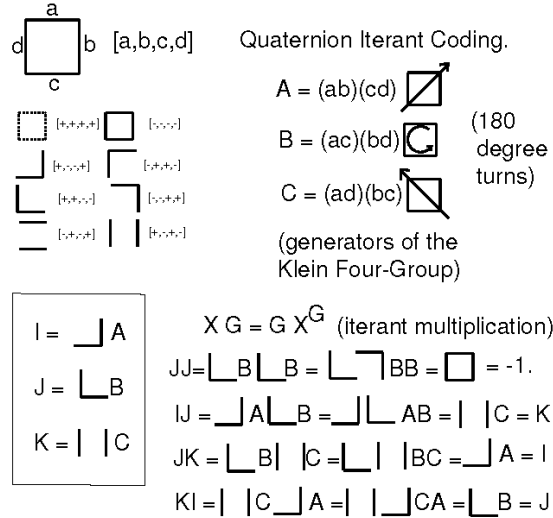


Figure 36: The 2D alphabet 4

Electromagnetism and much other physics can be written in quaternionic language. One can start with a Clifford algebra with generators e_1, e_2, e_3, e_4 with $(e_i)^2 = -1$ and distinct elements anti-commuting and construct spacetime algebra, the quaternions, and more. The iterant structure that we have hinted at here is part of a reformulation of the mathematics of matrix algebra that puts it into a temporal framework and a framework that respects the ubiquitous appearance of the symmetries of permutation groups. It is likely that in another generation of the RD concept, we shall include more about the role of symmetry. In this way, we have the beginnings of a relationship of RD structure and fundamental frameworks for physical theory.¹² All this said, we have made only a superficial connection between the spacetime algebra of the quaternions and the actions or operations of the 2D RD.

The iterant process is in back of the quaternion multiplication, where the symmetry group acts on the alphabetical letters. This could become part of an extension of RD operations. Then the RD would not just compare and describe. It would also interact with its own descriptions and change them by certain symmetry operations. This is one possibility for adding rules, but we do not yet have a clear picture of what extra structure can be added naturally to the very simple base with which we have started.

¹² For further details, see Schmeikal, "Basic Intelligence Processing Space"; Kauffman, "Iterants, Fermions, and Majorana Operators."

4. Distinctions, Distinctioning and Wolfram Automata

In this section, we make a comparison with the general structure of Wolfram line automata.¹³ The Wolfram automata use a very simple alphabet consisting of two letters (black and white, or 0 and 1). At every stage in the process, a distinction is applied to the eight possible states consisting of a square and its neighbors to the left and to the right. The distinction assigns 0 or 1 to each of these states, and the fate of the middle square in the next row is decided by that distinction. We see that these line automata are certainly RD automata, but that they are not strictly orthodox in our sense, in that the alphabet is not descriptive of all the local distinctions under consideration. The alphabet is simple, but the distinctions that can be made are complex. The result of this choice leads to a large and interesting body of phenomena.

In Figure 37, we see a depiction of the results of applying Wolfram Rule 126. As the reader can see, by comparison with Figure 1 and Figure 2, the overall pattern resulting from Rule 126 is essentially the same as that obtained from our 1D RD. The underlying structure of alphabet and distinction is different. This is a first example indicating the need for more detailed comparison between orthodox RD rules and cellular automata. We will leave such analysis for further work. In Figures 38 and 39 we illustrate Rule 110 and show how its iteration looks. It differs from Rule 126 in only one place. This de-symmetrization of Rule 126 results in very complex behavior. Here we are farther from the simple 1D RD. Rule 110 takes full advantage of the very simple alphabet of zero and one, and it uses an asymmetrical distinction on the set of eight triples of zeros and ones. The result is a very complex pattern of evolution and an automaton that has been proved to be Turing universal. One can certainly regard Rule 110 as a highly successful application of non-orthodox RD. We will return to this rule in a subsequent paper and examine it further in the light of RD structure.

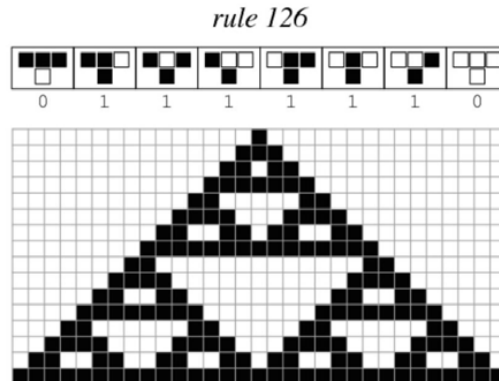


Figure 37: Wolfram Rule 126

¹³ Wolfram, *New Kind of Science*.

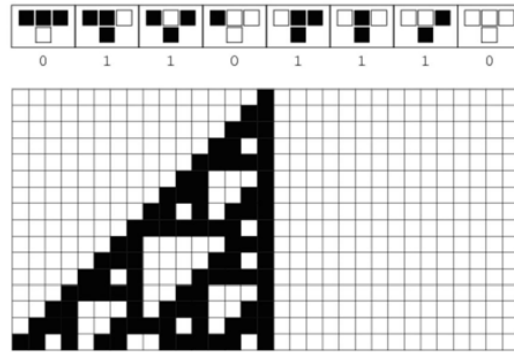


Figure 38: Wolfram Rule 110

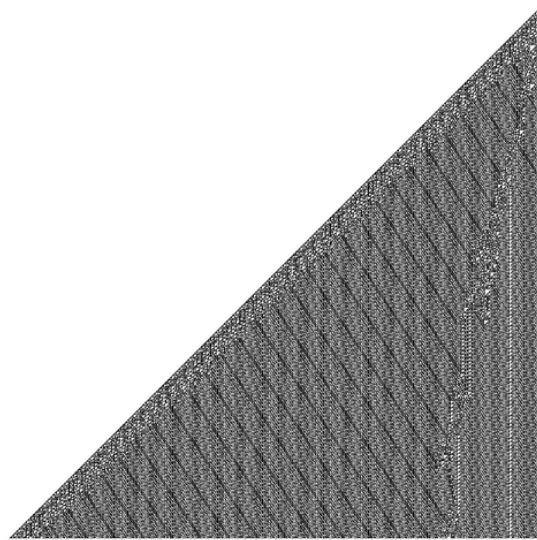


Figure 39: Wolfram Rule 110

5. The HighLife Replicator

This section is a comparison of patterns of the self-replicating element in the 1D RD and a very similar pattern in the much more complicated environment of the two-dimensional cellular automaton called HighLife, a variant of John Horton Conway's *Game of Life*. In HighLife, the environment is a rectangular lattice and each square is regarded as having eight neighbors. We could analyze an orthodox RD with an alphabet that generalizes the 16-letter alphabet to a $256 = 2^8$ letter alphabet for this geometry. This analysis is a future project for us. HighLife uses a simple binary rule. Each square in the lattice is either occupied (by a marker) or it is unoccupied (unmarked). We say that a square has n neighbors (where n is between 0 and 8) if n of its neighboring squares are occupied. The rule for HighLife is that an occupied square will survive (remain occupied) only if it has two or three neighbors. Otherwise it will become unmarked ("die"). An unoccupied square will become occupied (be "born") if it has three or six neighbors. In HighLife,

there is a remarkable, small configuration that can reproduce itself. It takes 12 steps for this replication process to take place. See Figures 40-52.

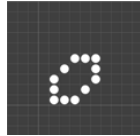


Figure 40: The HighLife Replicator

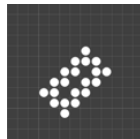


Figure 41: The HighLife Replicator

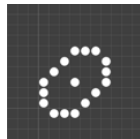


Figure 42: The HighLife Replicator

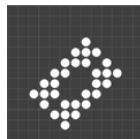


Figure 43: The HighLife Replicator

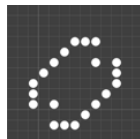


Figure 44: The HighLife Replicator

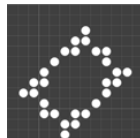


Figure 45: The HighLife Replicator

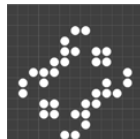


Figure 46: The HighLife Replicator

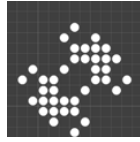


Figure 47: The HighLife Replicator

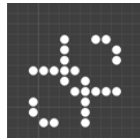


Figure 48: The HighLife Replicator

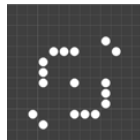


Figure 49: The HighLife Replicator

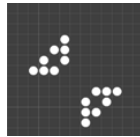


Figure 50: The HighLife Replicator

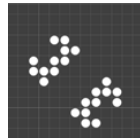


Figure 51: The HighLife Replicator



Figure 52: The HighLife Replicator

Quite remarkably, the pattern that these replicators follow is essentially the same as the pattern that is followed by the self-replicating element in the 1D RD. See Figures 53 to 60.

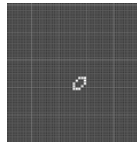


Figure 53: The 12-step HighLife replicator

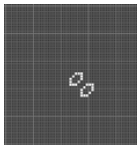


Figure 54: The 12-step HighLife replicator

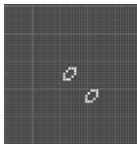


Figure 55: The 12-step HighLife replicator

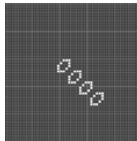


Figure 56: The 12-step HighLife replicator

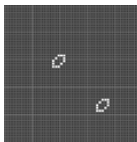


Figure 57: The 12-step HighLife replicator

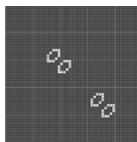


Figure 58: The 12-step HighLife replicator

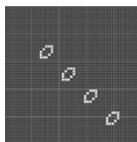


Figure 59: The 12-step HighLife replicator

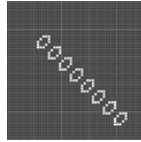


Figure 60: The 12-step HighLife replicator

6. RD and DNA

We begin this section with a review of material from the introduction to the paper. In this section, we describe one version RD process, and we show how it gives rise to a pattern of self-replication that is recognizable as a case of replication that we have called *DNA replication*.¹⁴

The rules for the RD process are very simple. We begin with an arbitrary, finite text string delimited by the character * at both ends. The RD process creates a new string from the given string by *describing the distinctions in the initial string*. Each character in the initial string is examined together with its left and right neighbors. Let LCR denote a character C with neighbors L and R. Then we replace C by a new character according to the following rules:

1. $C \rightarrow =$ if $L = C$ and $C = R$ (no distinction).
2. $C \rightarrow [$ if $L \neq C$ but $C = R$ (distinction on the left).
3. $C \rightarrow]$ if $L = C$ but $C \neq R$ (distinction on the right).
4. $C \rightarrow O$ if $L \neq C$ and $C \neq R$ (distinction on both the left and the right).
5. If C is adjacent to * change C to = (This is just a choice of boundary behavior).

See Figure 3 for the result of applying the RD process to a chosen text string.

In Figure 1, we showed the result of starting with a very simple text string. In this figure we do not print the character =, so that the resulting strings have empty space where this character would appear. As the reader can see, the string * =====]O[===== * has a long sequence of transformations under the RD process. The pattern]O[is replicated by the sequence below.

```

1  =====]O[=====
2  =====]OOO[=====
3  =====]O[=]O[=====

```

Remarkably, this self-replication has the same pattern as an abstract description of DNA replication. We explain this below in a separate section.

¹⁴ Kauffman, "Biologic"; Kauffman, "Self-Reference."

6.1 A Quick Review of the Pattern of DNA Replication

DNA consists of two strands of base-pairs wound helically around a phosphate backbone. It is customary to call one of these strands the Watson strand and the other the Crick strand. Abstractly, we can write

$$\text{DNA} = \langle W \mid C \rangle$$

to symbolize the binding of the two strands into the single DNA duplex. Replication occurs via the separation of the two strands via polymerase enzyme. This separation occurs locally and propagates. Local sectors of separation can amalgamate into larger pieces of separation as well. Once the strands are separated, the environment of the cell can provide each with complementary bases to form the base pairs of new duplex DNAs. Each strand, separated *in vivo*, finds its complement being built naturally in the environment. This picture ignores the well-known topological difficulties present to the actual separation of the daughter strands (see Figure 61). In this figure, we give some hints about the topological complexities that are not discussed here. Biologists discovered enzymes that cut and reconnect strands of DNA, resulting in the release of topological linking that would otherwise obstruct the separation of the newly produced strands of DNA. All this is subject to another discussion of its relationship with RD concepts.

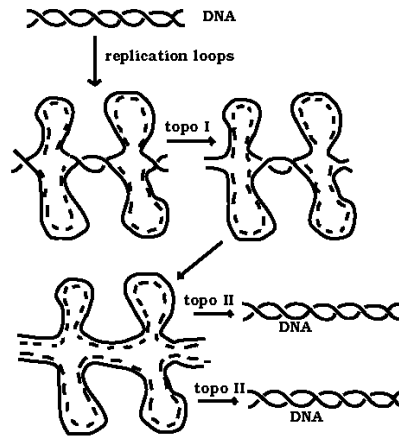


Figure 61: DNA Replication

The base pairs in the DNA sequence are AT (Adenine and Thymine) and GC (Guanine and Cytosine). Thus if

$$\langle W \mid = \langle \text{TTAGAATAGGTACGCG} \mid$$

then

$$\mid C \rangle = \mid \text{AATCTTATCCATGCGC} \rangle.$$

Symbolically we can oversimplify the whole process as

$$\langle W | + E \rightarrow \langle W | C \rangle = \text{DNA}$$

$$E + | C \rangle \rightarrow \langle W | C \rangle = \text{DNA}$$

$$\langle W | C \rangle \rightarrow \langle W | + E + | C \rangle = \langle W | C \rangle \langle W | C \rangle$$

Either half of the DNA can, with the help of the environment, become a full DNA. We can let $E \rightarrow | C \rangle \langle W |$ be a symbol for the process by which the environment supplies the complementary base pairs AG, TC to the Watson and Crick strands. In this oversimplification, we have cartooned the environment as though it contained an already-waiting strand $| C \rangle$ to pair with $\langle W |$ and an already-waiting strand $\langle W |$ to pair with $| C \rangle$.

In fact, it is the opened strands themselves that command the appearance of their mates. They conjure up their mates from the chemical soup of the environment.

The environment E is an identity element in this algebra of cellular interaction. That is, E is always in the background and can be allowed to appear spontaneously in the cleft between Watson and Crick:

$$\langle W | C \rangle \rightarrow \langle W | | C \rangle \rightarrow \langle W | E | C \rangle$$

$$\rightarrow \langle W | | C \rangle \langle W | | C \rangle \rightarrow \langle W | C \rangle \langle W | C \rangle.$$

This is the formalism of DNA replication.

We are now in a position to compare the formalism of the DNA replication with the RD replication.

$$1 \quad \text{=====]O[=====}$$

$$2 \quad \text{=====]OOO[=====}$$

$$3 \quad \text{=====]O[=]O[=====}$$

In the RD replication, we start with]O[in its RD environment. Matters of distinction of this entity from its surroundings lead to the production of]OOO[, and then we see that the identity of the internal O with its neighbors leads to the splitting]O[=]O[. There is no question that the basis of this replication is not the same as the DNA replication, but thematically, the two patterns are certainly related. The RD pattern is at a different level than the DNA pattern. In the RD replication, that environment for the symbol string is the larger symbol string. Thus it is only in the eyes of the observer of the RD that the entity]O[is distinguished and is seen as an actor against the background of declarations of identity =====. These declarations of identity are indeed equal to one another and so form an invariant background or void from which patterns arise in the presence of any difference. This is, in fact how our entity came into being.

AAAAAAAAAAAAAAAAABAAAAAAAAAAAAA

=====]O[=====

Our entity]O[is the first description of sameness on left, difference in middle, sameness on the right. The left and right icons] and [form a carapace for the indicator of difference O. Thus a bare difference of B from its equal neighbors A evolves by description, at once into a *proto-cell* with a carapace. It is this protocell that then undergoes mitosis in the next two rounds of description. The cell-division or mitosis is enabled by the production of new carapace (]OOO[→]O[=]O[) from within the cell. It is important to note that this production does not come from an inner mechanism of the cell, but rather from the global recursive/descriptive situation of these entities in the entire line of the RD structure. It is the influence of the surrounding void that makes all this happen in the course of recursive description and distinction. It is a fortuitous accident of working in one dimension that the carapace is seen in a left portion paired with a right portion, analogous to the two strands of the DNA. *At this condensed creation scenario, we find that the patterns of DNA replication, cell formation, and mitosis all appear at once in the first few steps away from a marking (B) in the void (of repeated As).*

For DNA replication, we can interpret the correspondence as:

1.] = Watson, [= Crick, O = backbone or binding.
2. RD action results in the opening of the backbone so that binding O is replaced by environment OOO.
3. RD action relative to the environment results in the placement of a new Watson and a new Crick. So we have the self-replication of]O[.

Note that there is another level at which we can think about this! Regard] and [as cell walls. Then we are witnessing not DNA reproduction, but *mitosis* itself! The little fellow]O[is a cell and we are watching how it reproduces in the line environment ===== of the void where there are no distinctions. The reader should now look again at Figure 3 and note the many appearances and interactions related to this elementary cell.

Of course the interpretations of backbone, strand, environment, and cell are different from what happens in the biology, but it is very interesting that the basic principles are similar.

Note how we get ==]OOOOO goes to ==]O[=== So actually the whole environment flips here. But it is contained in the above scenario. Everything that happens in RD is non-local, since a single event affects the whole string.

Perhaps it is clear to the reader that RD in the sense of this section is a potentially explosive topic that will grow to influence all the aspects of biology and computing. We believe that this is the case. The principle of [distinction/description in recursive process] applies at all levels of biology, cognition, information science, and computing.

7. Maturana, Uribe, and Varela and the Game of Life

Some examples from cellular automata clarify many of the issues about replication and the relationship of logic and biology. Here is an example due to Maturana, Uribe, and Varela.¹⁵ The ambient space is two dimensional and in it there are “molecules” consisting of “segments” and “disks” (the catalysts; see Figure 62). There is a minimum distance between the segments and the disks (one can place them on a discrete lattice in the plane). And “bonds” can form with a probability of creation and a probability of decay between segment molecules with minimal spacing. There are two types of molecules: “substrate” (the segments) and “catalysts” (the disks). The catalysts are not susceptible to bonding, but their presence (within say three minimal step lengths) enhances the probability of bonding and decreases the probability of decay. Molecules that are not bonded move about the lattice (one lattice link at a time) with a probability of motion. In the beginning, there is a randomly placed soup of molecules with a high percentage of substrate and a smaller percentage of catalysts. What will happen over the course of time?

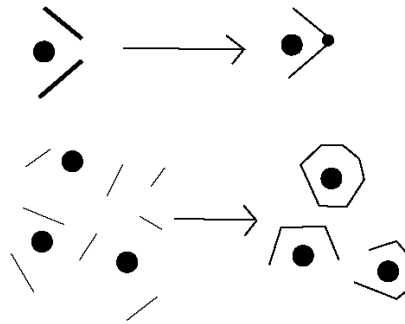


Figure 62: Proto-Cells of Maturana, Uribe, and Varela

In the course of time, the catalysts (which are basically separate from one another due to lack of bonding) become surrounded by circular forms of bonded or partially bonded substrate. A distinction (in the eyes of the observer) between inside (near the catalyst) and outside (far from a given catalyst) has spontaneously arisen through the “chemical rules.” Each catalyst has become surrounded by a proto-cell. No higher organism has formed here, but there is a hint of the possibility of higher levels of organization arising from a simple set of rules of interaction. *The system is not programmed to make the proto-cells.* They arise spontaneously in the evolution of the structure over time.

8. Conway Life

One might imagine that organisms could be induced to arise as the evolutionary behavior of formal systems. There are difficulties, not the least of which is that there are

¹⁵ H. R. Maturana, R. Uribe, and F. G. Varela, “Autopoiesis: The Organization of Living Systems, Its Characterization and a Model,” *Biosystems* 5 (1974): 7-13. See also F. J. Varela, *Principles of Biological Autonomy* (New York: North Holland Press, 1979) for a global treatment of related issues.

nearly always structures in such systems whose probability of spontaneous emergence is vanishingly small. A good example is given by another automaton – John H. Conway’s Game of Life. In Life, the cells appear and disappear as marked squares in a rectangular planar grid. A newly marked cell is said to be born. An unmarked cell is dead. A cell dies when it goes from the marked to the unmarked state. A marked cell survives if it does not become unmarked in a given time step. According to the rules of Life, an unmarked cell is born if and only if it has three neighbors. A marked cell survives if it has either two or three neighbors. All cells in the lattice are updated in a single time step. The Life automaton is one of many automata of this type and indeed it is a fascinating exercise to vary the rules and watch a panoply of different behaviors.

For this discussion, we concentrate on some particular features. There is a configuration in Life called a glider (see Figure 63), which illustrates a series of gliders going diagonally from left to right down the Life lattice, as well as a glider gun (discussed below) that has produced them. The glider consists of five cells in one of two basic configurations. Each of these configurations produces the other (with a change in orientation). After four steps, the glider reproduces itself in form, but shifted in space. Gliders appear as moving entities in the temporality of the Life board. The glider is a complex entity that arises naturally from a small random selection of marked cells on the Life board. Thus the glider is a naturally occurring entity, just like the proto-cell in the Maturana-Urbe-Varela automaton.

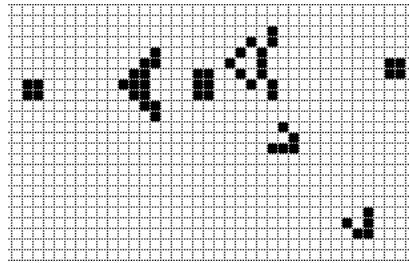


Figure 63: Glider gun and gliders

But Life contains potentially much more complex phenomena. For example, there is the glider gun (see Figure 63), which perpetually creates new gliders. The gun was invented by the Gosper Group, a group of researchers at MIT in the 1970s. It is highly unlikely that a gun would appear spontaneously in the Life board. Of course, there is a tiny probability of this, but we would guess that the chances of the appearance of the glider gun by random selection or evolution from a random state is similar to the probability of all the air in the room collecting in one corner. Nevertheless, the gun is a natural design based on forms and patterns that do appear spontaneously on small Life boards. The glider gun emerged through the coupling of the power of human cognition and the automatic behavior of a mechanized formal system.

Cognition is, in fact, an attribute of our biological system at an appropriately high level of organization. Cognition itself looks as improbable as the glider gun! Do patterns as complex as cognition or the glider gun arise spontaneously in an appropriate biological context?

There is a middle ground. If one examines cellular automata of a given type and varies the rule set randomly rather than varying the initial conditions for a given automaton, then a very wide variety of phenomena will present themselves. In the case of molecular biology at the level of the DNA there is exactly this possibility of varying the rules, in the sense of varying the sequences in the genetic code. So it is possible at this level to produce a wide range of remarkable complex systems.

9. Other Forms of Replication

Other forms of self-replication are quite revealing. For example, one might point out that a stick can be made to reproduce by breaking it into two pieces. This may seem satisfactory on the first break, but the breaking cannot be continued indefinitely. In mathematics, on the other hand, we can divide an interval into two intervals and continue this process ad infinitum. For a self-replication to have meaning in the physical or biological realm, there must be a genuine repetition of structure from original to copy. At the very least, the interval should grow to twice its size before it divides (or the parts should have the capacity to grow independently).

A clever automaton, due to Chris Langton, takes the initial form of a square in the plane. The square extrudes an edge that grows to one edge length and a little more, turns by ninety degrees, grows one edge length, turns by ninety degrees grows one edge length, turns by ninety degrees and when it grows enough to collide with the original extruded edge, cuts itself off to form a new adjacent square, thereby reproducing itself. This scenario is repeated as often as possible, producing a growing cellular lattice (see Figure 64).

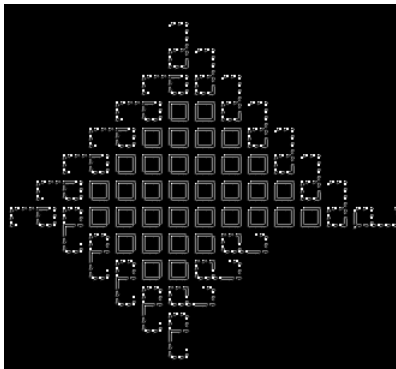


Figure 64: Langton's automaton

The replications that happen in automata such as Conway's Life are all really instances of periodicity of a function under iteration. The glider is an example where the Life game function L applied to an initial condition G yields $L^5(G) = t(G)$ where t is a rigid motion of the plane. Other intriguing examples of this phenomenon occur. For example, the initial condition D for Life shown in Figure 65 has the property that $L^{48}(D) = s(D) + B$ where s is a rigid motion of the plane and $s(D)$ and the residue B are disjoint sets of marked squares in the lattice of the game. D itself is a small configuration of eight marked squares fitting into a rectangle of size 4 by 6. Thus D has a probability of $1/735471$ of being chosen at random as eight points from 24 points.

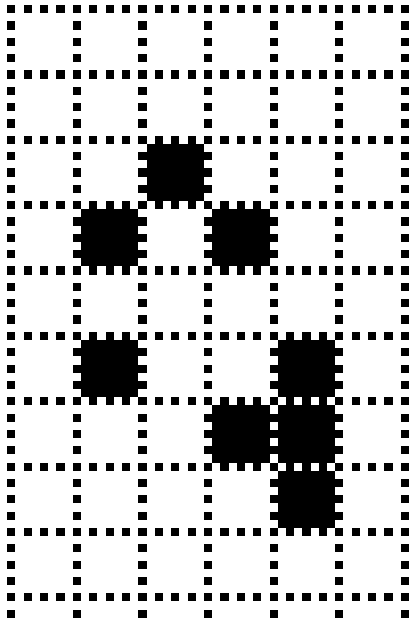


Figure 65: Condition D with geometric period 48

Should we regard self-replication as simply an instance of periodicity under iteration? Perhaps, but the details are more interesting in a direct view. The glider gun in Life is a structure GUN such that $L^{30}(\text{GUN}) = \text{GUN} + \text{GLIDER}$. Further iterations move the disjoint glider away from the gun so that it can continue to operate as an initial condition for L in the same way. A closer look shows that the glider gun is fundamentally composed of two parts P and Q such that $L^{10}(Q)$ is a version of P and some residue, and such that $L^{15}(P) = P^* + B$, where B is a rectangular block, and P^* is a mirror image of P, while $L^{15}(Q) = Q^* + B'$ where B' is a small non-rectangular residue. See Figure 66 for an illustration showing the parts P and Q (left and right) flanked by small blocks that form the ends of the gun. One also finds that $L^{15}(B + Q^*) = \text{GLIDER} + Q + \text{Residue}$. This is the internal mechanism by which the glider gun produces the glider.

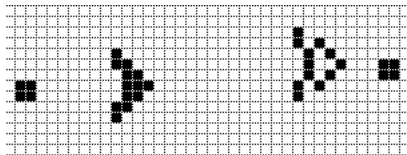


Figure 66: P (left) and Q (right) compose the glider gun

The extra blocks at either end of the glider gun act to absorb the residues that are produced by the iterations. Thus the end blocks are catalysts that promote the action of the gun. Schematically the glider production goes as follows:

$$P + Q \rightarrow P^* + B + Q^*$$

$$B + Q^* \rightarrow \text{GLIDER} + Q$$

whence

$$P + Q \rightarrow P^* + B + Q^* \rightarrow P + \text{GLIDER} + Q = P + Q + \text{GLIDER}.$$

The last equality symbolizes the fact that the glider is an autonomous entity no longer involved in the structure of P and Q. It is interesting that Q is a spatially and time shifted version of P. Thus P and Q are really copies of each other in an analogy to the structural relationship of the Watson and Crick strands of the DNA. The remaining part of the analogy is the way the catalytic rectangles at the ends of the glider gun act to keep the residue productions from interfering with the production process. This is analogous to the enzyme action of the topoisomerase in the DNA.

The point about this symbolic or symbiological analysis is that it enables us to take an analytical look at the structure of different replication scenarios for comparison and for insight.

There are a number of variants of Conway Life. We have earlier in this paper discussed HighLife and its self-replicator, whose pattern is a direct relative to the self-replicator in the 1D RD. Kauffman discussed another variant of Conway Life¹⁶ and denoted it by the name 7-Life in that paper. The generative rule for 7-Life is B37/S23, meaning that an empty square gives birth to a marked square if it has either three neighbors or seven neighbors, and a marked square survives to the next generation if it has either two or three neighbors. Conway Life is defined by the distinction B3/S23. In Conway Life, one has gliders that occur naturally and we have discussed the glider gun that emerged from a design interaction with computer experiments using Conway Life. However, 7-Life behaves differently from Conway Life. There are still naturally occurring gliders, but relatively small initial configurations tend to behave dynamically, interacting via the gliders to produce self-sustaining, slowly growing configurations. These configurations can eventually give birth to more complex self-reproducing entities.¹⁷ The entity that emerges, usually after thousands of iterations, is more complex (a pair of mirror-imaged configurations) than the glider, but by our experience, not so improbable as never to emerge! This leads to the question of the possibility and probability of the emergence of complex structures, analogous to biological structures, in the forward history of an RD automaton. We mention the cases of non-orthodox RD and experiments of this kind since structurally, all these automata do operate recursively on the basis of distinctions made at each step. The variants of Conway Life and the Wolfram automata are all very simple instances of RD where the basic language is binary and there is only one distinction made at each step.

10. Laws of Form

In this section, we discuss a formalism of G. Spencer-Brown in his book *Laws of Form*,¹⁸ which is often called the calculus of indications. This calculus is a study of mathematical foundations with a topological notation based on one symbol, the mark

¹⁶ Kauffman, "Reflexivity and Eigenform."

¹⁷ Ibid.

¹⁸ G. Spencer-Brown, *Laws of Form* (New York: Julian Press, 1969).

⌈.

This single symbol represents a distinction between its own inside and outside. The mark is seen as making a distinction, and the calculus of indications is a calculus of distinctions, where the mark refers to the act of distinction. The mark is self-referential and refers to its own action and to the distinction that is made by the mark itself. Spencer-Brown is quite explicit about this identification of action and naming in the conception of the mark, and by the end of the book he reminds the reader that “the mark and the observer are, in the form, identical.” We make this discussion here because it is important to trace the origins of the idea of distinction that is so central to the present paper.

The concept of distinction as used in Laws of Form is very close to that used implicitly in set theoretic mathematics. There the fundamental distinction is represented by set brackets (the act of collecting into a set) and the empty set { } is the first distinction.

In the calculus of indications, the mark can interact with itself in two possible ways. The resulting formalism becomes a version of Boolean arithmetic, but fundamentally simpler than the usual Boolean arithmetic of 0 and 1 with its two binary operations and one unary operation (negation).

Remarkably, the calculus of indications provides a context in which we can say exactly that a certain logical particle, the mark, can act as negation *and* can interact with itself to produce itself.

The mathematics in Laws of Form begins with two laws of transformation about these two basic expressions. Symbolically, these laws are:

1. Calling $\lrcorner\lrcorner = \lrcorner$
2. Crossing $\lrcorner\lrcorner =$

The equals sign denotes a replacement step that can be performed on instances of these patterns (two empty marks that are adjacent or one mark surrounding an empty mark). In the first of these equations, two adjacent marks condense to a single mark, or a single mark expands to form two adjacent marks. In the second equation, two marks, one inside the other, disappear to form the unmarked state indicated by nothing at all. That is, two nested marks can be replaced by an empty word in this formal system. Alternatively, the unmarked state can be replaced by two nested marks. These equations give rise to a natural calculus, and the mathematics can begin. For example, *any expression can be reduced uniquely to either the marked or the unmarked state.* The following example illustrates the method:

$$\begin{aligned} \overline{\overline{\lrcorner\lrcorner\lrcorner\lrcorner}} &= \overline{\overline{\lrcorner\lrcorner\lrcorner}} = \overline{\lrcorner\lrcorner} \\ &= \overline{\lrcorner} = \lrcorner \end{aligned}$$

The general method for reduction is to locate marks that are at the deepest places in the expression (depth is defined by counting the number of inward crossings of boundaries needed to reach the given mark). Such a deepest mark must be empty and it is either surrounded by another mark, or it is adjacent to an empty mark. In either case, a reduction can be performed by either calling or crossing.

Laws of Form begins with the following statement. "We take as given the idea of a distinction and the idea of an indication, and that it is not possible to make an indication without drawing a distinction. We take therefore the form of distinction for the form." Then the author makes the following two statements (laws):

1. The value of a call made again is the value of the call.
2. The value of a crossing made again is not the value of the crossing.

The two symbolic equations above correspond to these statements. First, examine the law of calling. It says that the value of a repeated name is the value of the name. In the equation

$$\neg\neg = \neg$$

one can view either mark as the name of the state indicated by the outside of the other mark. In the other equation

$$\overline{\neg} =$$

the state indicated by the outside of a mark is the state obtained by crossing from the state indicated on the inside of the mark. Since the marked state is indicated on the inside, the outside must indicate the unmarked state. The Law of Crossing indicates how opposite forms can fit into one another and vanish into nothing, or how nothing can produce opposite and distinct forms that fit one another, hand in glove. The same interpretation yields the equation

$$\neg = \neg$$

where the left-hand side is seen as an instruction to cross from the unmarked state, and the right hand side is seen as an indicator of the marked state. The mark carries a double meaning. It can be seen as an operator, transforming the state on its inside to a different state on its outside, and it can be seen as the name of the marked state. That combination of meanings is compatible in this interpretation.

From the calculus of indications, one moves to algebra. Thus

$$\overline{\overline{A}}$$

stands for the two possibilities

$$\overline{\neg} = \neg \leftrightarrow A = \neg$$

$$\overline{\neg} = \leftrightarrow A =$$

In all cases we have

$$\overline{A} = A$$

By the time we articulate the algebra, the mark can take the role of a unary operator

$$A \rightarrow \overline{A}$$

But it retains its role as an element in the algebra. Thus begins algebra with respect to this non-numerical arithmetic of forms. The primary algebra that emerges is a subtle precursor to Boolean algebra. One can translate back and forth between elementary logic and primary algebra:

1. $\neg \leftrightarrow T$
2. $\overline{\neg} \leftrightarrow F$
3. $\overline{A} \leftrightarrow \sim A$
4. $AB \leftrightarrow A \vee B$
5. $\overline{A} \overline{B} \leftrightarrow A \wedge B$
6. $\overline{A} B \leftrightarrow A \Rightarrow B$

The calculus of indications and the primary algebra form an efficient system for working with basic symbolic logic.

By reformulating basic symbolic logic in terms of the calculus of indications, we have a ground in which negation is represented by the mark *and* the mark is also interpreted as a value (a truth value for logic) and these two interpretations are compatible with one another in the formalism. At this point the reader can appreciate what has been done if he or she returns to the usual form of symbolic logic. In that form we see that

$$\sim \sim X = X$$

for all logical objects (propositions or elements of the logical algebra) X. We can summarize this by writing

$$\sim \sim =$$

as a symbolic statement that is outside the logical formalism. Furthermore, one is committed to the interpretation of negation as an operator and not as an operand. The calculus of indications provides a formalism where the mark (the analog of negation in that domain) is both a value and an object, and so can act on itself in more than one way.

The mark as linguistic particle is its own anti-particle. It is exactly at this point that physics meets logical epistemology. Negation as logical entity is its own anti-particle. In our view, the world and the formalism we use to represent the world are not separate. The observer and the mark are (formally) identical. A path is opened between logic and physics.

The visual iconics that create via the half-boxes of the calculus of indications a model for the mark as logical particle can also be seen in terms of cobordisms of surfaces (see Figure 67). There the boxes have become circles and the interactions of the circles have been displayed as evolutions in an extra dimension, tracing out surfaces in three dimensions. The condensation of two circles to one is a simple cobordism between two circles and a single circle. The cancellation of two circles that are concentric can be seen as the right-hand lower cobordism in this figure with a level having a continuum of critical points where the two circles cancel. A simpler cobordism is illustrated above on the right where the two circles are not concentric, but nevertheless are cobordant to the empty circle. Another way of putting this is that two topological closed strings can interact by cobordism to produce a single string or to cancel one another. Thus, a simple circle can be a topological model for the mark, for the fundamental distinction.

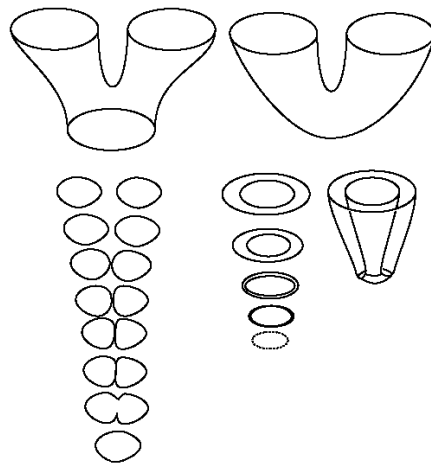


Figure 67: Calling, crossing, and cobordism

We are now in a position to discuss the relationship between logic and quantum mechanics. We go below Boolean logic to the calculus of indications, to the ground of distinctions based in the phenomenology of distinction arising with the emergence of concept and percept together, in the emergence of a universe in an act of perception.

Here we find that the distinction itself is a logical particle that can interact with itself to produce itself, but can also interact with itself to annihilate itself. The fundamental state is a superposition of these two possibilities for distinction. We are poised between affirmation of presence and the fall into an absence that we cannot know. This superposition is likely not yet linear in the sense of the simple model of quantum theory. Nevertheless, it is at this source, the place of arising and disappearing of awareness, that we come close to the quantum world in our own experience. As always, this experience is known to us in ways more intimate than the reports of laboratory experiments. It is the uniqueness of every experience, of every distinction. There can be no other one. There is only this and this and this yet again.

Nevertheless, one can go on and consider quantum states related to the aforementioned logical particle. Crossing this boundary into quantum theory proper, one finds that topology and physics come together in this realm, and there is a complex possibility of much new physics to come and a new basis for quantum computing.¹⁹ It will take more thought and a sequel to this paper to begin to sort out the relationships between quantum theory and RD at the level of this form of epistemology.

Remark. In Laws of Form we can express $XOR(A, B) = A^B = B^A$ by the formula

$$A^B = B^A = \overline{A|B} \overline{A|B}$$

Note that if B is marked, then

$$A^{\neg} = \overline{A|}$$

Thus the operation of XOR is the action of the mark itself. We can regard diagrammatic circuits such as we used in Figure 6 as applications of the mark in the form of the XOR operation above. In this way, the apparently awareness-dependent operations of the Laws of Form shift to the automatic discrimination capabilities of computer circuits and the forms of RD can be seen as written in the language of the calculus of indications. These points of view inform each other circularly.

11. Commentary

Here is a collection of remarks and insights into RD that come from conversations between the authors of this paper over a number of years.

1. Joel: When distinction-making is applied to a pattern there is a new pattern that is comprised of the variety of distinctions recorded. Thus, a new pass of distinction-making can be applied to the pattern of distinctions, and this kind of a process can repeat itself recursively, indefinitely.

¹⁹ For further details, see Louis H. Kauffman. "Knot Logic and Topological Quantum Computing with Majorana Fermions," in *Logic and Algebraic Structures in Quantum Computing and Information: Lecture Notes in Logic*, ed. J. Chubb, Ali Eskandarian, and V. Harizanov (Cambridge: Cambridge University Press, 2016), 223-336.

2. Joel: I had made a discovery (mathematical in nature) of processes of RD (which is not patentable *per se*), and then invented a physical embodiment that performs these processes.
3. Joel: The sensing of gradients (chemical concentrations, nutrients, etc.) in bacteria is well established and demonstrable. These are elements of distinction-making at very primitive levels. It is much harder to demonstrate recursive distinction-making in bacteria, because these are more abstract operations. It can be done however with live neuron circuits, and about 250K separate us now from results of such a demo.

Eshel Ben-Jacob proposed that recursive distinction-making may be easier to demonstrate in genetic/immunological systems and it would also be much cheaper than the work planned with neurons. I am waiting for more details. At any rate, Eshel's program is all interrelated, with recursive distinction-making being a unifying theme.

4. Joel: I tend to think in terms of sensory-driven cognition that is constructed bottom-up, beginning with Stage 1 – sensory distinctions; and proceeds to Stage 2 – indefinite recursion that starts out from Stage 1 and builds up successive layers of distinctions-of-distinctions. It is unlikely that these two low-level stages involve awareness. A working hypothesis is that some sort of awareness emerges from the primitive Stages 1 and 2 towards a level that you identified as Type 1. So, basically I tend to think of your Type 1 as an epiphenomenon that arises from Stages 1 and 2. [Type 1 for LK is a distinction that comes simultaneously with an awareness of that distinction.] I believe (actually I have shown) that Stages 1 and 2 are mechanizable. A missing link, of course, is the transition from Stages 1 and 2 to your Type 1. I am very sympathetic to constructivist dispositions and the place of human beings in the order of things. I agree that thought thinking itself is all we have got but I see no contradiction in proposing that thought processes have their ultimate genesis in pre-cognitive and pre-aware primitive processes of sensory-driven RD.
5. Joel: Spencer-Brown has been very seductive to a lot of people and rightfully so. For most of us, drawing a distinction is a cognitive act that is performed by a full-blown human being. Spencer-Brown, of course, represents a distinction by some sketching of circles on a piece of paper by a human. I don't object to this! That's how much of mathematics is done. Scribbling of some symbols, sometimes in reference to some drawings of geometric or topological configurations. But doubts linger. Is it possible to entertain a situation where distinctions are drawn by acts that are short of being cognitive? And if this is possible, where is the observer, the self? And what constitutes the other? What will happen to the expected dynamics of "I and Thou"? Will there emerge a "becoming"? Becoming of what? It seems utterly futile to concoct a scenario of distinction-making at a level that is well below a cognizing person. (And what's left of constructivism if the cognizing person is dissolved to his sensory modalities?) [LK: Note that Spencer-Brown never discusses how distinctions arise but always discusses distinctions that are accompanied by an awareness or an observer.]

Well, the thing is this. Sensory modalities, all of them, must make local distinctions in certain features (e.g., intensities) in signals that impinge on them. It has been studied in great detail in visual perception, beginning with the retina. Photoreceptors in the retina make local distinctions of light intensities that impinge on the retina. (Absent this, capacity for local distinctions amounts to blindness.) This local distinction-making is accomplished by comparisons that ultimately cause firing/non-firing. These processes involve certain physiological/biochemical processes, in conjunction with massive neural circuits. The above type activity is clearly pre-cognitive, involuntary, and (with sufficient abstraction) can be accomplished by computing machines as well.

The essence of my patent document is RD (in one-dimension; but it is motivated directly by RD in 2D, which operates on 2D digital imager; 2D RD is abstracted from local distinction-making at the retinal level, as worked out by Weisel and Hubel in the early 1960s).

I recently sketched for the history of my ideas (beginning in the early 1960s) and how these are embedded in the patent document, including the basis for *fantomarks* and their streaks.

I think that the singular contribution of my particular RD processes is operationalizing the process of recursion on distinction-making. For it gives precise and detailed trace of what it entails, including an emergent dialectics, circularity, and so on.

To be sure, other people have talked about recursive distinction-on-distinction (notably Maturana, in the context of his much higher-level “*languaging*”), but it should be clear that my RD is at a precognitive level, is mechanizable, and affords a thorough examination of its emergent properties.

6. Joel: I noticed that thing – the hypothetical distinction (or contingent distinction) that hasn’t actually been made. It exemplifies the potency of distinction, even if not acted upon. These are the wonders of distinction, actual, virtual, potential, contingent, and hidden, to name only a few types. Now, when these are compounded via recursion – watch out!
7. Joel: I have no objection to make a (provisional) distinction between the kind of distinction in RD automata and the Maturana and Varela kind of distinction. In itself, this act of distinction between two distinctions is a good example of what RD automata typically do. I think that, in the end, we’ll mutually discover that the distinction between the two kinds of distinctions will gradually dissolve.

Here is a succinct description of the roles of distinction in RD automata: In RD automata, we have two basic elements that involve notions of distinction.

1. An element of distinction-making. This element involves acts of distinguishing (verb) and is a process.

2. The results of distinctioning are a collection of distinctions; where a distinction is a product, object (noun).

Usually these objects form a pattern of distinctions (the pattern as a whole is also an object) that is subject to further acts of distinctioning.

Thus process and products alternate, recursively, where both process and products involve notions that relate to distinction.

The process element involves distinction-making; and the product element is a pattern of objects, referred to as distinctions. (Each such distinction is a local, fragmentary boundary that records the result of prior acts of distinctioning.)

It is crucial to understand that the alternation between process/product is recursive and indefinite in duration; also, that such indefinite recursion is guaranteed to drive the process into circularity. This, as a whole, represents the notion of RD in RD automata. (It is called BIP in the patent.)

The RD automata model is motivated by natural vision. The initial stages are motivated by the retina, and the rest of the recursive process is postulated to take place in the lateral geniculate nucleus (LGN) and the visual cortex proper.

In recent years, some researchers in advanced techniques in neural circuits (not artificial neural nets, but rather actual, live neural tissue) have entertained the hypothesis that a certain version of RD automata takes place in normal brain tissue activity.

8. Joel: This is to systematize RD by dimension.

* 1D – This is the case that is documented in the patent. It was pre-dated by the 2D case. A neighborhood comprises three elements, where a central element has two neighbors. There are exactly four combinations of relationships between an element and its two neighbors, representable by four ideographs, as described in the patent.

* 2D – This is the case that relates to image processing; it goes back to 1964. A neighborhood (Moore neighborhood) is comprised of nine elements, where a central element has eight neighbors. There are exactly 256 combinations of relationships between a central element and its eight neighbors. These are representable by 256 ideographs.

The 2D case can be decomposed into a network of 1Ds. For comparison, John Conway's Game of Life is also run on a Moore neighborhood but has only two states (as compared to 256 [!] states in the Game of RD). The richness and complexity of Game of Life is well known. Imagine the complexity of this 2D RD game.

* 3D – A neighborhood is comprised of 27 elements, where a central element has 26 near neighbors. There are exactly 2^{26} (i.e., 67,108,864) combinations of

relationships between a central element and its 26 near-neighbors. Clearly, I didn't investigate this case. Instead, I retreated to the 0D case; see below.

* 0D – This is the case where RD starts with a single speck against the void. It yields the scenario of the baryon octet, as described elsewhere.

9. Joel and Lou: Your comment is interesting. There are RD processes that are uniquely in the purview of human observers. There are certain RD processes that can be performed by automata, and there may also be RD processes in nature. The challenge is to integrate all three types into an encompassing framework whose unifying theme is RD processing. As to experimenting with CA, there are obviously untold numbers of possible CA, some of which have extremely interesting behaviors. In RD we focus on a singular cellular automaton, the one CA whose rule is recursive distinction-making. Once we grasp that distinction-making is a unique operation (in regards to perception and cognition) we realize that we must focus on the particular class of RD automata, in preference to the other zillions of CA possibilities that are available for our consideration and entertainment. I submit that RD automata are the needles in the haystack of CA.
10. Lou: In programs that we design the initial automatic distinctions are distinctions that are put in by design. In the observation of such programs new distinctions arise for us, that can be used for further designs. But in nature, it is not obvious how those structures that we are calling distinction operators have arisen. We do not imagine that they occur by design. We do not imagine that they were ideas in the mind of a designer. I am very aware of this issue. as I have experimented at other levels with cellular automata and have seen how by varying rule structures one can find extraordinary recursive structures that one would never have imagined. Our relationship with our own constructions and with nature is complex.
11. Joel: Transdistinction operates on patterns of raw sensory data to produce a first pass of local distinction-making in such patterns. Further processing is relegated to higher centers in the nervous system. (For example, this is essentially what the retina does [in part] in vision.) This first pass is relatively easy to accomplish by computing devices. Thus, impairment in a sensory organ can be overcome by using such prosthetic devices. The next issue, of course, is how to connect the output of the prosthetic device to higher centers. In vision, for example, a connection needs to be done to the optic nerve, or directly to the lateral geniculate nucleus, from which the normal vision pathways would be followed to the visual cortex. Assuming that such devices will become reality, would it modify our notion of the observer? Namely, a human observer so equipped would initiate his or her observation by an automatic device that does distinction-making. So, there you have it – a hybrid of human/machine in a long sequence of distinction-making; some automatic and some human-based.
12. Joel: Yes, quids and quods seem to be generalized notions of containers/extainers.
[Lou: Extainers have the formalism $E = \succ$ while containers have the formalism $C =$

<>.] Extainers are open to interaction from the outside. Containers are closed forms not likely to interact. But note that

$$EE = ><>< = > C <$$

and

$$CC = <><> = < E >.$$

Thus an extainer interacts with an extainer to produce a container, and a container interacts with a container to produce an extainer. We can distinguish between containers and extainers by allowing containers to move freely (commute) with other elements. Then

$$EE = > C < = C >< = CE$$

and we see that C can be the catalyst for self-replication. And if we regard the extainer as the environment, then the movement

$$<> \rightarrow < E >$$

can be seen as our earlier abstraction of the emergence of Watson and Crick strands from the environment. We obtain the self-replication of DNA type:

$$<> \rightarrow < E > \rightarrow <><>.^{20}$$

Inasmuch as quids and quods come about literally out of nowhere (they are byproducts of RD that operates on arbitrary initial unspecified things, including fantomarks), their natural algebra may be significant.

Quids and quods (discrete/continuous) are self-organized. They enter into an elaborate dance that is not choreographed by external manipulation. The dance has classical dialectical patterns.

Replication is part of the game. There are at least two types of replication:

1. For RD with fixed boundaries, there is guaranteed circularity. Thus a whole bunch of strings are periodically replicated. These happen to be 4-letter strings with certain complementarity properties. Close enough to DNA, but not quite the same.
2. For RD with shifting extainers (such as in the Baryon Octet scenario), there is replication of patterns via self-similarity in the trace. In effect, a basic pattern reappears periodically.

²⁰ See Kauffman, "Iterants, Fermions, and Majorana Operators"; Kauffman, "Biologic"; and Kauffman, "Self-Reference, Biologic and the Structure of Reproduction" for more about extainers and containers.

All in all, I propose to consider the algebra of quids/quods (which extends your [Lou K.] notions of containers/extainers) somewhere at the foundations of your marvelous edifice.

There is an example of this in Figure 2 on page 11 of my paper "Steganographic Representation of the Baryon Octet in Cellular Automata." This is an RD that starts out with a first arbitrary distinction. Focus on lines 1 thru 8.

'0' is like your container that fuses <> together. It may contain at most one thing. There is a notion of extended container, written: < * * >, which may contain a bunch of things. (It shows in Figure 2 as [==== =]: C is an element of quids, and the extended container is a quod, as defined in the patent document.) Now, make the following substitutions in Fig. 2:

0 is C
] is >
[is <
= is *

Lines 1 thru 8 will look like this:

>C<
>CCC<
>C<*>C<
>CCCCCCC<
>C<*****>C<
>CCC<***>CCC<
>C<*>C<*>C<*>C<
>CCCCCCCCCCCCCCCC<

and you can continue thru line 16 and beyond. Within that 16-line diagram you can identify 10 configurations that look like this:

>C<
CCC
<*>

Those 10 configurations are self-organized similarly to the Pythagorean Tetractys.²¹ Those configurations allow us to uncover the configuration of the baryon octet that is embedded therein.²²

Thus the physical interpretation of > and < are up and down quarks and * is a strange quark.

Let's recoup what we're doing. We start out with a first distinction and apply RD to it. We develop the trace of a cellular automaton that does RD. Within that trace we discover the Pythagorean Tetractys, within which we discover the eight particles of the baryon octet expressed in terms of their constituent quarks. Note: There ought to be a link to SU(3), which still eludes me.

13.Lou: Clearly we have just begun this study. There is much more to come.

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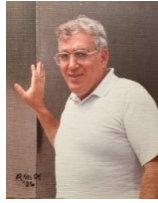
About the Authors: **Louis H. Kauffman** is Professor of Mathematical Physics and Cybernetics at the University of Illinois, Chicago. He has degrees from MIT and Princeton. He has 170 publications. He was the founding editor for the *Journal of Knot Theory and its Ramifications* and he writes a column entitled "Virtual Logic" for the journal *Cybernetics and Human-Knowing*. He was president of the American Society for Cybernetics from 2005-2008. He introduced and developed the Kauffman Polynomial. He was the recipient of the 2014 Norbert Wiener Award of the American Society for Cybernetics.



Joel D. Isaacson has pioneered in RD Cellular Automata since the 1960s. Recursive Distinctioning was rooted in studies relating to the analysis of digitized biomedical imagery. He utilized NASA's computing facilities at the Goddard Space Flight Center in Greenbelt, MD for the initial stages of this research. His research has been supported over the years by DARPA, SDIO, NASA, ONR, USDA and a good number of NIH institutes. He is Professor Emeritus of Computer Science, Southern Illinois University, and Principal Investigator for the IMI Corporation.

²¹ Isaacson, "Steganographic Representation," Figure 3, p. 12.

²² Ibid., Figure 7, p. 16.



Editors' Notes: The Board of Directors of Kepler Space Institute (KSI) and the editors of the *Journal of Space Philosophy* take pride in providing the publication platform for Dr. Joel D. Isaacson and Dr. Louis H. Kauffman to inform the public on the current status of RD. That term is the scientific description of "Nature's Cosmic Intelligence" (Joel Isaacson, *Journal of Space Philosophy* 1, no. 1 [Fall 2012]: 8-16) that Dr. Isaacson discovered in 1964. Since that date he has been the lead scientist and scholar in researching this information stream phenomenon that Dr. Bernd Schmeikal – whose supporting paper is also in this Special Science Issue of the *Journal of Space Philosophy* – has called "a universal creative system." Dr. Isaacson described RD in April 2011 as "a finding that is advanced as a law of nature, perhaps on the par of gravity." Over the past two years, Dr. Louis Kauffman, one of America's most distinguished mathematicians and physicists, has worked intensively with Dr. Isaacson to create this latest scientific explanation for the world. For further information on RD, see homepages.math.uic.edu/~kauffman/RD.html. **Bob Krone and Gordon Arthur.**

Basic Intelligence Processing Space

By Bernd Schmeikal

Abstract

This paper investigates a universal creative system. Originally, this was referred to by its creator as an *autonomic string manipulation system*. Forty years ago, it was capable of such important operations as tetracoding (TTC) and binary *basic intellector processing* (BIP). After going deeper into the set of possible transformations, in both a sequential and a parallel manner, Joel Isaacson and Louis Kauffman had brought this down to the essential action of *Recursive Distinctioning* (RD). Considering the dual process of *antecursive conflation*, we can unpack a given creation – like a page taken out from a libretto – and trace it back to some initial headlines. We unpack the creation of our Minkowskian space-time with its geometric algebra, and show how it can be made a material representative of a BIP. So, after clarifying a few issues of ideographing, we state that the processes BIP, *digital image processing*, and TTC by RD, which were invented and investigated by Joel Isaacson, are real articulations of the natural space-time with its material systems of interacting particles. That is to say, our universe may be a representation of Isaacson's system, and entertainingly, with his US Patent specification 4,286,330, it seems he has patented creation.

Keywords: Universal creative systems, autonomic string manipulation, intellector processor, Recursive Distinctioning, antecursive conflation, Minkowski algebra, image processing, primordial space creation, retinoid cortical space, standard model of particle physics.

Prologue

This paper discusses a universal, dialectic, intelligent process, whose creator, 40 years ago, endowed it with some clumsy-looking names like *autonomic string-manipulation system* and *basic intellector processing* (BIP). This humble, dynamic system, featuring Hegel's triadic phenomenology of mind,¹ is very creative. It is now capable of reckoning up words in the retinoid visual cortex, and acts of creation, almost like the spill over from an unbounded living universe. In his early work, Joel Isaacson has used eleven *string-manipulation operations* to study the properties of his creation. Then, he realised that the tetracoding (TTC) and BIP were most important operations anyway. Kauffman and Isaacson have studied this system for a long time. They saw that there is a unary procedure on strings that is necessary for understanding creational processes, namely *Recursive Distinctioning* (RD).

My task is now to clarify some iconic coding issues brought in by the original operation of *ideographing* and to couple the creational process to theoretical physics. After all, we formerly based our synchronous template of the Minkowski space algebra and

¹ G. W. F. Hegel, *The Phenomenology of Mind*, trans. J. B. Baillie (London: George Allen & Unwin, 1966); G. W. F. Hegel, *The Phenomenology of Spirit*, trans. Terry Pinkard, terrypinkard.weebly.com/phenomenology-of-spirit-page.html (accessed February 13, 2016).

equations of motion on this operation; the standard model of high energy physics (HEPhy) and its symmetries are emerging results from such a recursive, scrambled, and entangled iterative process. I will show in exact terms how and why Isaacson 's self-organizing string manipulation system creates synchronous iconic memories of objects in Minkowski algebra. Since I have found that the symmetries of space-time algebra are essentially those of the standard model of physics, it is reasonable to ask how an iteration may accidentally create an octet of the symmetric unitary group $SU(3)$. Indeed, both, the Clifford algebra of Minkowski space and the standard model of particle physics are deep and dynamically stable properties of some peculiar system of linear processing. Now, BIP and digital image processing (DIP) produce two important representations of results in our material world. The first is given by hardware implementations, the second by material assemblies in our familiar, relativistic space-time. It is interesting to note that the autonomic string-manipulation system, published with US Patent 4,286,330 on August 25, 1981, was a continuation of an application filed in April 1976. It referred to a related document having same title, filed in December 1975.² However, the "Dialectical Machine Vision" report came much later, namely in July 1987. It contained the ideographs of DIP cells combined with the hesitant denotation of "the 'alphabet' of the visual cortex," and some cryptic sentences on page 35: "For many years I have resisted describing DIP in 'neural' terms."

So, one would conclude that the retinoid ideographs were much younger than the autonomic string-manipulation system. But that would be wrong. On October 25, 2015, Joel wrote me, "I discovered the 16 icons a long time ago, in 1964. I was developing image processing techniques to analyse 2D digitised images. Two neighbourhoods have been available to me: 8-cell Moore neighbourhood and 4-cell von Neumann neighbourhood. The first led to 256 icons, where the 16 icons (that you now work with) were a subset. And the second led to 16 icons that are exactly identical with your 16 icons." There were some graphics with a description mailed to Louis Kauffman in 2012 when Louis was visiting at the Isaac Newton Institute in Cambridge. A special digitised radiograph was scanned by the film input into digital automatic computer and fed directly into the core memory of an IBM 7094 at NASA's Goddard Space Flight Center, Greenbelt, Maryland. It was thereafter analysed with the aid of a first tiny alphabet of 256 icons based on the 8-cell Moore neighbourhood and materialised by a Stromberg-Carlson 4020 microfilm recorder from NASA.

The computerised ideograph recorded fragments of the local boundaries of images. A little square has eight neighbouring squares. Each of them may be black or white. This makes a total of 256 combinations, the possible marks of a Moore filter. One goes from pixel to pixel, or in a font printout from character to character, and identifies the neighbourhood as one of those 256. Clearly, it is just as informative if one counts the von Neumann neighbourhoods. That makes no difference. However, the alphabet is restricted to the 16 necessary letters. Then, some of these icons were used, but their meaning as generators in algebraic modules was not yet explained. What is an advantage, mathematically, is that those 16 provide a basis of four, which can

² Application Ser. No. 674,658, filed April 7, 1976, and Disclosure Document entitled "Autonomic String-Manipulation System," No. 045773, filed on December 29, 1975.

immediately be identified within the geometric algebra of space-time. After all, logic connectives have a plane markedness. But then it was considered a virtue to proceed in a line with a minimal neighbourhood system to avoid perceptron-type models because of the disrepute attached to them,³ since Minsky and Papert had shown up their limitations. It was preferable to proceed with linear strings and a minimal neighbourhood, not in 3-line arrays or similar planar domains.

Components of Autonomic Intelligence

When, in April 1976, Joel Isaacson wrote a continuation of his autonomic string manipulation system, he wanted to design something extremely primitive, something that would take off with almost no processing capability, no memory or internal description of outer configurations, and that would process input regardless of type, classification, and complexity “in a blind, purposeless, and primitive fashion.” These were the words Isaacson chose in his 1981 patent specification.⁴ He must have felt that such a stupid processor, being aware of the presence of just a few nearest neighbours, would nevertheless, by the runtime, disclose what to us may appear as untermiated intelligence. To be precise, the *intellector process*, as it was called then, is not unbounded, but has definite boundaries; yet, its intelligence develops in an open-ended fashion. The beauty of the intelligent forms it (re)creates is continuing indefinitely, and while it creates various forms of remembrance, some gilded, some just surprising, it shows to us what Hegel once meant by his phenomenology of mind, with its reappearing and self-reproducing cycles driven by contradiction and synthesis. What once seemed so strange and superhuman, almost inhuman, all of a sudden turns out to be a self-evident feature of a most simple form of process driven by contact.

The basic processor manipulates strings of symbols or marks such as, say, the word 23f3f23trxff223. But we might just as well consider linear sequences of pixels with a grey level, or colour value, as inputs. What we need for manipulation is awareness or an identification of a given character. This character can be prehended or sensed or recorded by human beings, in which case it is also correlated with semiotic terms such as sign, icon, pictogram, index, token, ideogram, ideograph, and so forth. So we have sensual and cognitive attributes guiding a string. These perceptions and annotations allow us to refer to such a character as an objective element or a datum object.⁵ An element that cannot be perceived in that way is referred to as a *fantomark*. Strings containing fantomarks are called fantomark strings. We may denote it as basic intelligence if the processor acts according to the neighbourhood. So, we have fundamental operations acting on strings, reading them line by line. But we also have

³ Joel D. Isaacson, “Dialectical Machine Vision, Applications of Dialectical Signal-Processing to Multiple Sensor Technologies,” Report prepared for the Strategic Defense Initiative Organization (Arlington, VA: Office of Naval Research, 1987), 35.

⁴ Joel D. Isaacson, “Autonomic string-manipulation system,” US Patent No. 4286330 A, priority 1976, (publication date 1981), 8, patft.uspto.gov/netacgi/nph-Parser?Sect2=PTO1&Sect2=HITOFF&p=1&u=/netahtml/PTO/search-bool.html&r=1&f=G&l=50&d=PALL&RefSrch=yes&Query=PN/4286330. This was a continuation of application Ser. No. 674,658, filed April 7, 1976 with no cross-references to related applications, and a single relevant reference to a related disclosure document entitled Autonomic String-Manipulation System, No. 045773, filed on December 29, 1975.

⁵ Isaacson, “Autonomic string-manipulation system,” columns 3-4.

parallel, quasi-synchronous perception of the nearest neighbours. The identification of symbols in the immediate neighbourhood of an observed character in the linear sequence of marks in a string allows us to introduce techniques of RD, namely those in Isaacson's patent, and the procedures of *streaking* and TTC. Consider the closed string

2 3 f 3 f 2 3 t r x f f 2 2 3 having definite length 15

If we read from left to right and identify distinctions from neighbours, streaking brings on

0 0 0 0 0 0 0 0 0 1 0 1 0 0 with a zero appended to the end of a code sequence, and

A A A A A A A A A B C B C A are the tetracoded characters of the string.

For example, consider the second character in the original string, namely 3. It has left neighbour 2 and right neighbour *f*, both distinct from 3, hence tetracode *A*. Next, *f* has left and right neighbours 3, both distinct from *f*, hence tetracode *A*. We obtain a code mark *B* at location ... *x f f* ... because the left neighbour is distinct from *f* and the right neighbour is not distinct. Finally, we obtain mark *C* at location ... 2 2 3

The operations of streaking and TTC, on a topological basis of the nearest neighbours in linear sequences of marks, represent the most relevant methods in BIP. Clearly, it is possible to encode a tetracoded string by TTC. Isaacson gave the beautiful example of self-referential TTC of the word BEGINNING and the sentence SEE PERFECT CYCLE.⁶ We add the ENDING on the right of Table 1.

⁶ Isaacson, "Autonomic string-manipulation system," 3.

Table 1 a, b: Re-entering Strings to the Operations of TTC

(a)	1	2	3	4	5	6	7	8	9	(b)	1	2	3	4	5	6
00	B	E	G	I	N	N	I	N	G	00	E	N	D	I	N	G
01	A	A	A	A	B	C	A	A	A	01	A	A	A	A	A	A
02	B	D	D	C	A	A	B	D	C	02	B	D	D	D	D	C
03	A	B	C	A	B	C	A	A	A	03	A	B	D	D	C	A
04	A	A	A	A	A	A	B	D	C	04	A	A	B	C	A	A
05	B	D	D	D	D	C	A	A	A	05	B	C	A	A	B	C
06	A	B	D	D	C	A	B	D	C	06	A	A	B	C	A	A
07	A	A	B	C	A	A	A	A	A							
08	B	C	A	A	B	D	D	D	C							
09	A	A	B	C	A	B	D	C	A							
10	B	C	A	A	A	A	A	A	A							
11	A	A	B	D	D	D	D	D	C							
12	B	C	A	B	D	D	D	C	A							
13	A	A	A	A	B	D	C	A	A							
14	B	D	D	C	A	A	A	B	C							
15	A	B	C	A	B	D	C	A	A							
16	A	A	A	A	A	A	A	B	C							
17	B	D	D	D	D	D	C	A	A							
18	A	B	D	D	D	C	A	B	C							
19	A	A	B	D	C	A	A	A	A							
20	B	C	A	A	A	B	D	D	C							
21	A	A	B	D	C	A	B	C	A							
22	B	C	A	A	A	A	A	A	A							

The original invention, BIP, is concerned with unary operations on single strings, the operands. But it also allows for context-sensitive rewriting rules. Simultaneous application of rewriting rules to all characters in the operand is denoted as parallel. Sequential operation within the operand from the leftmost to the rightmost is called sequential. Figures 4A to E of the patent show how the array of tetracode strings can be transposed onto icons, broken lines, and streaks which can, again, be re-entered into a TTC procedure. Operations that are neither parallel nor strictly sequential are possible, and they are referred to as scrambled.

Notice that although we work with a minimal neighbourhood involving two neighbours only, we obtain fourfoldness through the four code-letters *A, B, C, D*. In image processing with constant line length, preserving parallel connection of lines, two neighbours with four letters pack the same information as four neighbours with two letters, that is, a von Neumann neighbourhood.

Ideographing

Linear Iconic Single Strings

As we obtained a code mark, *B*, say, at location ... *x f f* ... because the left neighbour was distinct from *f* and the right neighbour was not distinct, and we obtained a *C* mark at location ... *2 2 3* ..., it was easy to insert icons: □ for *B* and □ for *C*. Considering the *A* – no neighbour identical with the mark – as somehow isolated, we can substitute the *A* with □. The *D* indicated identical left and right marks. Hence, it seems good to indicate that opening towards both sides by the icon □. In this way, every single string can be rewritten as an iconic word. For instance, we obtain the following for the first three lines.⁷

⁷ Isaacson, "Autonomic string-manipulation system," 3, Figure 4b.



In this case, the operands are single strings and the lines are vertically closed by the horizontal bars of the icons.

Parallel Three-Line Processing with von Neumann Neighbourhoods

Ideographing sequences of single strings by four icons, □, ⊐, ⊑, ⊒, is straightforward and intuitively appealing. Yet, there are certain restrictions when it comes to interpreting the meaning with respect to the geometry of space-time. Namely, there exists a specific alphabet, which I called LICO (abbreviating *linear iconic calculus*), having 16 icons with an algebraic basis of four elements. If we want to incorporate space-time processing, we have to consider a peculiar scrambling. We have to consider a parallel processing of three single strings, the first, in a way, representing some internal past of the run-time, while the third is a future-string result. Consider the alphabet

(1)

and a Hegelian cycle with strings numbered 11 to 22 from Table 1a. Rewriting those lines gives Figure 1:

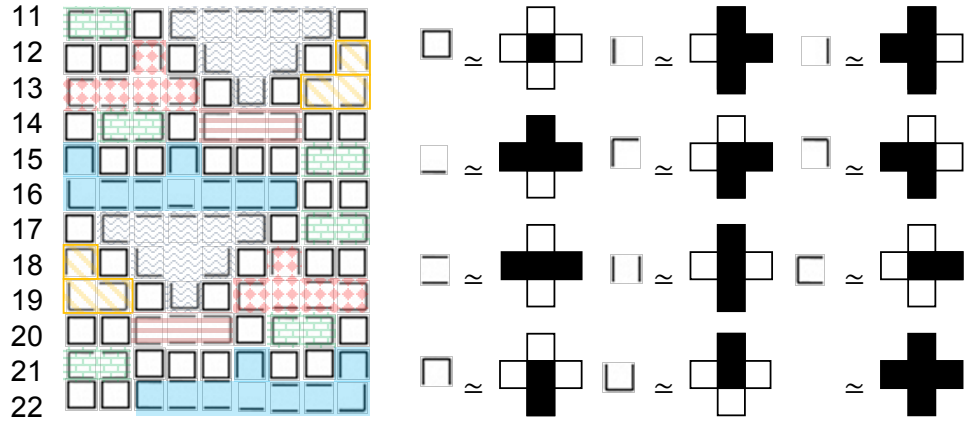


Figure 1: Three-line processing and 12 of 16 icons with corresponding von Neumann neighbourhoods.

Note that lines 11-16 are palindromic with lines 17-22.

The Electronic Circuits of the Intellector

In about 1982, the field of cellular automata (CA) started to take off, and by 1985, Isaacson succeeded in merging BIP and DIP with CA. While DIP seems to disclose higher complexity than BIP, the creational properties of the BIP must not be

underestimated. In the second part of his preface to the report, Isaacson has listed some of these: *autonomic mode of processing, autonomic error correction, autonomic mode of 3-level memory, dialectical patterns, autonomic syllogistic inferences, limit cycles or attractors (Hegelian cycles), autonomic generations of palindromes, and complementarity of 4-letter strings*. The most important processing unit carries out an iterative triunation of the streak of a given string and its successors. This amounts to a Hegelizing of the process by an electronic circuit denoted as the *intellector* (Figure 2), which essentially operates on binary sequences of given length.

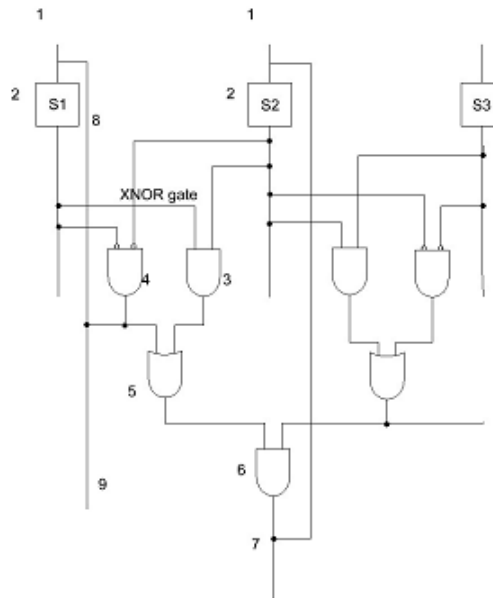


Figure 2: Basic circuitry of the intellector processor.

What was important for my own work was the appearance of the XNOR gates, the first from the left formed by Components 3, 4, and 5, because they represent a processing unit vector of the Clifford algebra of space-time. We shall come to this in a while. Isaacson described the important detail relating to XNOR gates and the circuit in Figure 7 of the patent. There are three binary signals (0 or 1) for A, B, and C at the input ports of s_1, s_2, s_3 . If all three signals are 0 or all three signals are 1, then the output is 1; otherwise, the output is 0. The logic expression describing the switching is given by $(A \text{ XNOR } B) \text{ AND } (B \text{ XNOR } C)$. This describes the operation *triunation* in the patent, and also Wolfram Rule 129. These are identical, but *triunation* predates Rule 129 by many years. Put in tandem, we get the combinatory circuitry of the whole of Figure 7 from the patent. As for oscillators, to realise the oscillators, we hold A and C fixed at 0 or at 1. If we set A and C at 0, we can start with $B_{(0)} = 0$ or $B_{(0)} = 1$.

For $B_{(0)} = 0$ we get the sequence:

$$B = 0, 1, 0, 1, 0, 1, \dots \quad (2)$$

For $B_{(0)} = 1$, we obtain

$$B = 1, 0, 1, 0, 1, 0, \dots$$

So, the smallest recursive triunation behaves like unitary oscillators, not unlike Kauffman's oscillatory sequences of the I and J.⁸ It turns out that these sequences can represent another generating unit of the Minkowski algebra. To understand the operations and dynamics of the autonomic string-manipulation system and the DIP, it is necessary to study the papers of Joel Isaacson. For the present, I just wish to refer to a few facts that concern the next section.

- there are signals;
- they are perceived as fourfold by TTC;
- they are rewritten by ideographs;
- they can be streaked;
- they are processed by serially connected XNOR gates;
- they begin ignorant, with almost no processing capability and no memory or internal description of outer configurations, and they process inputs in a blind, purposeless, and primitive fashion.

Real-World Components

Space-time, as a cognitive reality, represents a synchronous template, which I have constructed in order to coordinate real events. However, as a physical reality, space-time is an intelligent processing of energy. Years ago, when I tried to understand the relation between space-time and the standard model of HEPHY, it was not yet so clear that this processing had its own intelligence. But it was already evident that the central event under investigation was a processing of energy. It was known that space-time, be it explained by Euclidean space and separate time, or by relativistic, compound space-time, was connected with the concept of symmetry. In the simplest scenario, it would be natural to endow a Dreibein or a cube with an octahedral symmetry O_h or with a Bravais lattice, and, clearly, such an octahedral crystal – a diamond – would bring about a scattering of energy and of the observable degenerate energy levels. Could it be possible that space-time was responsible for the emergence of multiplets of elementary particles and energy spectra? These were the important questions, then. First, one had to identify the object that was worth being denoted as a mathematical agent of real space-time. Next, one had to find out about its symmetries. I felt that the symmetries of matter, the HEPHY standard, were essentially given by the symmetries of space-time. This led to a thirty-year endeavour, trying to conserve essential knowledge and at the same time to break away from the mainstream. It ultimately led to the book, *Decay of Motion – The Anti-Physics of Space-Time*.⁹ But the first breakthrough was published

⁸ Louis H. Kauffman, "Space and Time in Computation, Topology and Discrete Physics," In *Proceedings of the Workshop on Physics and Computation – PhysComp '94*, November 1994 (Dallas: IEEE Computer Society Press, 1995), 44-53.

⁹ Bernd Schmeikal, *Decay of Motion-The Anti-Physics of Space-time* (New York: Nova, 2014).

only in 1996 in a book about Clifford algebra, after I conversed with Pertti Lounesto.¹⁰ This was “The Generative Process of Space-Time.”¹¹

A picture arose in which several new ideas came together. First, it became clear that the complex matrices of the symmetric unitary group $SU(3, \mathbb{C})$ were elements of the matrix algebra $Mat(4, \mathbb{C})$, which represented the complexified Clifford algebra $\mathbb{C} \otimes Cl_{3,1}$. It could be that the standard model of HEPHY represented a space-time group rather than an auxiliary gauge group. In “Minimal Spin Gauge Theory,”¹² I investigated this alternative in greater detail, building on a preliminary inquiry of Roy Chisholm concerning “Unified Spin Gauge Theories and the Tetrahedral Structure of Idempotents.”¹³ It seemed to me that nature did not identify direction in such a definite way as we do in our laboratories. There was some basic uncertainty that disappeared in the stable arrangement of matter. Today, these features can be recognised as indicative of ignorance like that in the generative processes designed by Joel Isaacson.

Autonomic Intelligence

- Signals, marks, or polarised characters appear in diachronic succession.
- Processing is run in a blind, purposeless, and primitive fashion.
- No memory or internal description of outer configurations.

Physics

- Field quantization, fermions, condensates, and collapsing wave-functions are observed.
- Quantization is proceeding ignorant.
- No memory of the coding of outer configurations of coordinate base units.

Suppose, we had a triangle such as the one given by our image of a Euclidean Dreibein with three unit vectors (Figure 3).

¹⁰ Pertti found out that I was just about to rediscover Clifford algebra. I investigated the multivector groups of the Pauli algebra, that is, the Clifford algebra of the Euclidean 3-space. This algebra is generated by the three-dimensional Euclidean space, but can itself be considered as a vector space having dimension $2^3 = 8$. Surprisingly, the Minkowski space was a subspace of this orthogonal space generated by the Pauli matrices. I saw, then, what some had already known for a long time, that the four Dirac matrices used in theoretical physics also generated such a linear space of multivectors. This could be the 16-dimensional Clifford algebra of the Minkowski space, endowed with an indefinite metric $\{1,3\}$, what we denote as $Cl_{1,3}$, or it could be the Clifford algebra generated by the space in the opposite metric, the Lorentz metric, namely $Cl_{3,1}$. The latter has a 4×4 matrix representation, with real entries only. It is called Majorana algebra after Ettore Majorana, who first investigated nuclear weak decay with such real tools of differential geometry.

¹¹ Bernd Schmeikal, “The Generative Process of Space-Time and Strong Interaction – Quantum Numbers of Orientation,” in *Clifford Algebras with Numeric and Symbolic Computations*, ed. R. Ablamowicz, P. Lounesto, and J. M. Parra (Boston: Birkhäuser, 1996), 83-100.

¹² Bernd Schmeikal, “Minimal Spin Gauge Theory – Clifford Algebra and Quantumchromodynamics,” *Advances in Applied Clifford Algebra* 11, no. 1 (2001): 63-80.

¹³ J. S. R. Chisholm, “Unified Spin Gauge Theories of the Four Fundamental Forces,” in *Clifford Algebras and their Applications in Mathematical Physics*, ed. A. Micali et al. (Dordrecht: Kluwer, 1992), 363-70; J. S. R. Chisholm, “Tetrahedral Structure of Idempotents of the Clifford Algebra $Cl_{3,1}$,” in *Clifford Algebras and their Applications in Mathematical Physics*, ed. A. Micali et al. (Dordrecht: Kluwer, 1992), 27-32.

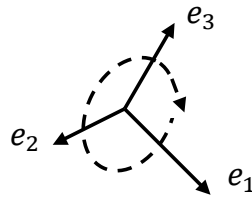


Figure 3: Rotation as recoding.

Consider a rotation that turns e_1 into e_2 , e_2 into e_3 , and e_3 into e_1 . Such a rotation, in a pure informatics sense, where nothing is known about such things as continuous motion, can be conceived as mere recoding. Mathematically, such a movement is represented by a permutation cycle (1, 2, 3). Now, consider that the process of nature in the deepest layer of dynamic phenomena cannot make those distinctions that we, the observers, are ready to make within the stratum of macrophysics. That could mean that the oscillators of field quantization, just like all the other movements we conceive of, do not distinguish between different base units of the Clifford algebra. I compile all such movements as *quantum motion* instead of as *quantum mechanics*. It might be that although the process of nature is ignorant of those differences the observers make within the macro-layers of the material world, it nevertheless brings those differences about. That would mean that the forces of nature, the ultraweak, the electromagnetic, the weak, and the strong interactions, give rise to the emergence of our concept of macroscopic space-time, in the form of a geometric Clifford algebra of the Minkowski space with its Lorentz metric, clearly, by making use not only of matter as space-time, but also of our brain cells, which somehow must incarnate this space-time as an inner neuronal arrangement. It seemed somewhat complicated to verify this idea, and I had to go slowly, step by step.

In Minimal Spin Gauge Theory, there were investigations of the relations between the orientation symmetries and the $SU(3)$, the action of reflections determined by the tetrahedral idempotent lattices. But, then, sociologically, it seemed, something had dazed us. The global reverberations and the anthropophobia before, during, and after the world wars had led to considerable rejections of insight and knowledge. These *social dislocations* probably had more important consequences for science than our later correcting measures of *quantum deformation*. Einstein had established an inner distance to quantum theory, and only lately had he realised the importance of Minkowski's work. Galina Weinstein confirmed what Gerhard Frey had said to me: "After he had received assistance from his friend, Marcel Grossmann, in late spring 1912, he found the appropriate starting point for a generalization"¹⁴ in terms of Minkowski's approach to space-time. He began to use the *line element* invariant under the Lorentz group. How come Einstein was side-tracked? Far off the beaten track of quantum mechanics, he began to describe the gravitational field by a metric tensor field. But if the properties of the space-time could describe gravitation, the ultraweak interaction, why could it not just as well, and even from the outset, describe the occurrence of quantum

¹⁴ Galina Weinstein, *Genesis of General Relativity – Discovery of General Relativity*, arxiv.org/ftp/arxiv/papers/1204/1204.3386.pdf (accessed February 4, 2016).

numbers of motion? I had to begin a back calculation. The following one is one of the many small, but important stages that had to be carried out to reach the aim. Particles should create their own space-time and HEPHY symmetries. If there existed the above fundamental uncertainty of quantum motion, we first had to investigate the algebraic object that carried out the transpositions of line elements in the basis of the Clifford algebra of the Minkowski space. This discrete group of 1,152 graded elements was found and was denoted as the *reorientation group* of the geometry $Cl_{3,1}$. It is a hyperoctahedral group generated by 24 multivectors having the form $s_{\chi k} = Id - 2f_{\chi k}$ ($\chi = 1, \dots, 6; k = 1, \dots, 4$) where the $f_{\chi k}$ are idempotents, primitive in the algebra $Cl_{3,1}$ having six colours χ and four indices determined by the basis of the Minkowski space. Accordingly, we take

$$\begin{aligned} f_1 &= \frac{1}{2}(Id + e_1)\frac{1}{2}(Id + e_{24}) & f_2 &= \frac{1}{2}(Id + e_1)\frac{1}{2}(Id - e_{24}) \\ f_3 &= \frac{1}{2}(Id - e_1)\frac{1}{2}(Id - e_{24}) & f_4 &= \frac{1}{2}(Id - e_1)\frac{1}{2}(Id + e_{24}) \end{aligned} \quad (3)$$

These primitive idempotents are Weyl's *erzeugende einheiten* for a linear subspace spanned by $ch_1 = span_{\mathbb{R}}\{Id, e_1, e_{24}, e_{124}\}$, and there are six such subspaces with positive definite metric $\{+ + + +\}$ in the Clifford algebra of the Minkowski space $\mathbb{R}^{3,1} \stackrel{\text{def}}{=} span_{\mathbb{R}}\{e_1, e_2, e_3, e_4\}$, having the indefinite signature of the Lorentz metric $\{+ + + -\}$. The quantities in the corners of Salomon's seal (Figure 4) span a commutative subspace, the *colour spaces* ch_{χ} . An idempotent primitive in $Cl_{3,1}$ represented by f_1 , endows ch_1 with a 1-norm. Take any $X = ald + be_1 + ce_{24} + de_{124} \in ch_1$ and verify that

$$f_1 X = (a + b + c + d)f_1 \text{ with 1-norm } L = a + b + c + d. \quad (4)$$

Therefore, we say that the $f_1 X$ provides an *eigenform* for a 1-norm.¹⁵ Take $L = a + b + c + d = 1$ to obtain $f_1 X = f_1$. It can be shown that each colour space ch_1, ch_2, \dots is isomorphic with the 4-fold real ring ${}^4\mathbb{R} = \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$.¹⁶ The proof goes as follows: Consider the idempotent

$$f = \frac{1}{2}(Id - e_1) \quad (5)$$

not primitive in the Minkowski algebra (but it is primitive in the Pauli algebra $Cl_{3,0}$) and the isospin

$$\Lambda_3 = \frac{1}{2}(e_{24} - e_{124}) \quad (6)$$

Both are elements in colour space ch_1 . For any natural number $n \in \mathbb{N}$ we verify the identities

¹⁵ Louis H. Kauffman, "Reflexivity and Eigenform – The Shape of Process," *Constructivist Foundations* 4, no. 3 (2009): 121-37; Heinz von Foerster, "Objects: Tokens for (Eigen-) Behaviors," in *Observing Systems*, Systems Inquiry Series (Seaside, CA: Intersystems Publications, 1981), 274-85.

¹⁶ Schmeikal, "Minimal Spin Gauge Theory," 63; Bernd Schmeikal, "Transposition in Clifford Algebra," in *Clifford Algebras – Applications to Mathematics Physics and Engineering*, ed. Rafal Ablamowicz (Boston: Birkhäuser, 2004), 351-72.

$$\Lambda_3^{2n} = f \text{ and } \Lambda_3^{2n-1} = \Lambda_3 \tag{7}$$

Therefore, the Clifford number Λ_3 represents a swap. The colourspace can now be decomposed into two ideals according to the equations

$$ch_1 = ch_1 f \oplus ch_1 \hat{f} = \mathcal{G}_1 \oplus \hat{\mathcal{G}}_1 \tag{8}$$

with main involuted \hat{f} , and spaces $\mathcal{G}_1 \stackrel{\text{def}}{=} \text{span} \{f, \Lambda_3\}$ and $\hat{\mathcal{G}}_1 \stackrel{\text{def}}{=} \text{span} \{\hat{f}, \hat{\Lambda}_3\}$. According to a theorem by Elié Cartan, all maximal abelian subalgebras of a semi-simple Lie algebra are mutually isomorphic. Further, the equations (7) imply that both \mathcal{G}_1 and $\hat{\mathcal{G}}_1$ are isomorphic with the small Clifford algebra $Cl_{1,0} := \{Id, e_1\} \simeq {}^2\mathbb{R} = \mathbb{R} \oplus \mathbb{R}$ – the double ring of real numbers. Therefore, due to Equation 8 we end up with a fundamental decomposition

$$ch_1 \simeq ch_\chi \simeq \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \text{ for each colourspace in the seal (see Figure 4).} \tag{9}$$

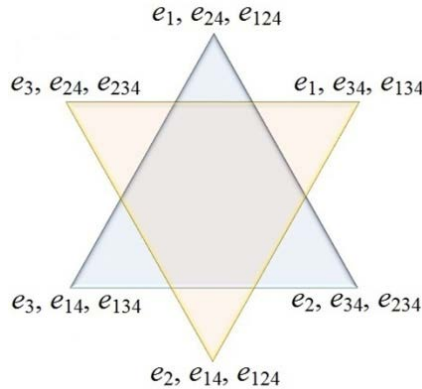


Figure 4: The seal of space-time – Cartan subalgebras of the motion-group.

Clearly, any colourspace can be spanned either by its orthogonal primitive idempotents or by its base units. If we consider the top of the seal, we represent the base units of ch_1 by the quadruples:

$$\begin{aligned} Id &= (+1, +1, +1, +1); e_1 = (+1, +1, -1, -1); \\ e_{24} &= (+1, -1, -1, +1); e_{124} = (+1, -1, +1, -1) \end{aligned} \tag{10}$$

With these numbers and Equation 3 we calculate the primitive idempotents $f_1 = (+1, 0, 0, 0)$; $f_2 = (0, +1, 0, 0)$; $f_3 = (0, 0, +1, 0)$; $f_4 = (0, 0, 0, +1)$; and \mathcal{G}_1 spanned by $f = (0, 0, +1, +1)$ and $\Lambda_3 = (0, 0, -1, +1)$. Now the story of the Minkowskian line elements, their metamorphosis, begins to become very interesting. In my books on primordial space, I had already investigated many important features of the HEPHY standard model space group. But now it had become possible to locate particles in a void without metric, and in such a way that they can represent the units of metric dynamic spaces themselves. In 2011, I heard Lou Kauffman speaking about eigenforms and

eigenvalues.¹⁷ That was on the occasion of the 100th birthday of Heinz von Foerster. Instead of using bivectors, Lou¹⁸ introduced the imaginary unit in the style of Rowan Hamilton,¹⁹ but developed the concept much further, by introducing the *iterant views* of dynamic systems for complex and quaternion arrays. Surprisingly, after having already restored the Dirac equation in this way in 1996, he now derived the discrete Schrödinger equation by the iterant algebra.²⁰ It did not take much more to seek a method to construct a geometric Clifford algebra with the aid of iterant algebra. This was first carried out in *Decay of Motion* and in two papers.²¹ Now it became possible to conceive of particles as fourfold strings of polarities. The affinity with Isaacson's streaks and time series of tetracodes became obvious.

See the analogy between the quad-locations²² and the ${}^4\mathbb{R}$ -representation of the base units of colourspace $ch_1 = \text{span} \{Id, e_1, e_{24}, e_{124}\}$. Due to the peculiar construction of the iterant algebra, we can identify the iterant views with units having different grades: a spatial unit, a space-time area, and a space-time volume.

$$e_1 := [+1, +1, -1, -1]; e_{24} := [+1, -1, -1, +1]; e_{124} := [+1, -1, +1, -1] \quad (11)$$

As we know, it is the trigonal transition among those iterants that brings out discrete colours, satisfying the unitary symmetry of the motion. On the other hand, the colourspace, being a commutative Cartan subalgebra of the $Cl_{3,1}$, is derived from the quaternion algebra by abstracting from the temporal order imposed on the iterants correlated with space ch_1 by the permutations φ, σ , and τ . In this sense, each colourspace $ch_\chi (\chi = 1, \dots, 6)$ turns out to be a *contemporised synchronous image* of the quaternion iterant temporal structure of relativistic quantum motion.

Space-Time Algebra from Its Logical Basis

Theorem:²³ The iterant algebra with four grades is isomorphic with the Clifford algebra $Cl_{3,1}$

Sketch of Proof: Consider the three real iterants e, f, g ; they are logic icons,

¹⁷ Louis H. Kauffman, "Eigenforms and Quantum Physics," *Cybernetics and Human Knowing* 18, no. 3-4 (2011): 111-21.

¹⁸ Louis H. Kauffman, "Imaginary Values in Mathematical Logic," in *Proceedings of the Seventeenth International Symposium on Multiple-Valued Logic* (Piscataway, NJ: IEEE, 1987), 282-89.

¹⁹ W. R. Hamilton, "Theory of Conjugate Functions, or Algebraic Couples; with a Preliminary and Elementary Essay on Algebra as the Science of Pure Time," *Transactions of the Royal Irish Academy* 1837, no. 17:293-422.

²⁰ Louis H. Kauffman, "Iterants, Fermions and Majorana Operators," in *Unified Field Mechanics, Natural Science Beyond the Veil of Spacetime — Proceedings of the IX Symposium Honoring Noted French Mathematical Physicist, Jean-Pierre Vigié*, Morgan State University, Baltimore, MD, November 16-19, 2014, ed. Richard L. Amoroso, Louis H. Kauffman and Peter Rowlands (Singapore: World Scientific Publishing, 2016), 1-32.

²¹ Bernd Schmeikal, "Four Forms Make a Universe," *Advances in Applied Clifford Algebra* 25, no. 1 (2015): 1-23, doi:10.1007/s00006-015-0551-z; Bernd Schmeikal, "Free Linear Iconic Calculus, AlgLog Part 1: Adjunction, Disconfirmation and Multiplication Tables," doi:10.13140/RG.2.1.2083.1841.

²² Schmeikal, "Four Forms Make a Universe," Table 12.

²³ This theorem is proved in Schmeikal, "Four Forms Make a Universe," as Theorem 18.

$$\sqsupset \simeq e = [+1, +1, -1, -1], \sqcap \simeq f := [+1, -1, -1, +1], \sqbar \simeq g := [+1, -1, +1, -1] \quad (11)$$

together with the permutation operators $\sigma := (1\ 2)(3\ 4)$, $\varphi := (1\ 3)(2\ 4)$, $\tau := (1\ 4)(2\ 3)$. These transpositions of characters are generated by iteration time t and tangle time η . Sequences are iterated by iteration time and by tangle time, and are applied to iterants of degree 4. The iterant time t is represented by a permutation 4-cycle $(1\ 2\ 3\ 4)$ and the tangle time by a 2-cycle $(1\ 2)$. These two generate the symmetric group S_4 . We have equations

$$\begin{aligned} \sigma[a, b, c, d] &= [b, a, d, c]\sigma \\ \varphi[a, b, c, d] &= [c, d, a, b]\varphi \\ \tau[a, b, c, d] &= [d, c, b, a]\tau \end{aligned} \quad (12)$$

Transpositions φ, τ, σ can be derived from iterant and tangle-time operators in this order

$$\begin{aligned} \varphi &= t^2 = (1\ 2\ 3\ 4)(1\ 2\ 3\ 4) = (1\ 3)(2\ 4), && \text{portrayed as cycles} \\ \tau &= \eta\varphi\eta = (2\ 1)((1\ 3)(2\ 4))(2\ 1) = (1\ 4)(2\ 3), && \text{palindromic operation} \\ \sigma &= \tau\varphi \end{aligned} \quad (13)$$

Now there exist nine possibilities to let any permutation operator act on the unit iterants. Among these nine products, there are six quaternions. Among those there are the three we already know from the analysis of quad locations. Three of the nine squared give the identity Id . The nine terms are $e\sigma, e\varphi, e\tau, f\sigma, f\varphi, f\tau, g\sigma, g\varphi, g\tau$. The idea of proving the theorem is challenged when we understand why among these nine we have six instead of three quaternions. That is, there are indeed two basic quaternion spaces in the Clifford algebra of Minkowski space, namely, a triple of *bivectors* $\{e_{12}, e_{23}, e_{13}\}$ with definite signature $\{-1, -1, -1\}$ and a further triple of time-like, quasi *thermodynamic* quaternions with different grades, the time-space quaternions $\{e_4, e_{123}, e_{1234}\}$. If we place these two sets of quaternions in parallel, we can see

$$\begin{array}{ccc} e_{12} & e_4 & e_{124} \\ e_{23} & e_{123} & \Rightarrow e_1 \\ e_{13} & e_{1234} & e_{24} \end{array} \quad (14)$$

how both quaternion groups, by Clifford multiplication, are carried to the *angular momentum Cartan subalgebra*, that is, to the colour space of the logic units. The Clifford product in each row gives a component of the first colour space, each of which squared gives the identity. Therefore, it is reasonable to assume, say, that the four quantities $\sqsupset, \sqbar, \varphi, \tau$ generate a geometric algebra that includes even more than just two sets of quaternions. This could be the Clifford algebra $Cl_{3,1}$ of the Minkowski space. To abbreviate the proof, let us factor in how the quantities $\sqsupset, \sqcap, \varphi, \tau$ interact.

Consider polarity strings e, f, g constituting the commutative algebra of a Klein-4 group; all the same the permutations σ, φ, τ satisfy the same algebra. The mixed products of

polarity strings and permutations commute or anti-commute.²⁴ For example, e commutes with σ , but f anticommutes with σ :

Use $e = [+1, +1, -1, -1]$, $\sigma := (1\ 2)(3\ 4)$, and (12): $\sigma[a, b, c, d] = [b, a, d, c]\sigma$, to get $\sigma e = (1\ 2)(3\ 4)[+1, +1, -1, -1] = [+1, +1, -1, -1]\sigma = e\sigma$; use $f := [+1, -1, -1, +1]$, $\sigma := (1\ 2)(3\ 4)$ and rule (12): $\sigma[a, b, c, d] = [b, a, d, c]\sigma$, to get $\sigma f = (1\ 2)(3\ 4)[+1, -1, -1, +1] = [-1, +1, +1, -1]\sigma = -f\sigma$; likewise, use $g := [+1, -1, +1, -1]$ and σ to verify $\sigma g = -g\sigma$; and so on until we get to $\tau g = -g\tau$. The result of exterior multiplication gives us the following representation of the Clifford algebra of Minkowski space $Cl_{3,1}$

$$\begin{array}{llll}
 Id & e_1 = e & e_2 = \varphi & e_3 = \tau f \\
 e_4 = f\varphi & e_{12} = e\varphi & e_{13} = g\tau & e_{14} = \varphi g \\
 e_{23} = \sigma f & e_{24} = f & e_{34} = -\sigma & e_{123} = \sigma g \\
 e_{124} = g & e_{134} = -\sigma e & e_{234} = -\tau & e_{1234} = \tau e
 \end{array} \tag{15}$$

We verify the signature of the Minkowski space, but first of all its Cartan subalgebra using the XNOR:

$$\begin{array}{l}
 e_1 e_1 = e^2 = [+1, +1, -1, -1](\equiv)[+1, +1, -1, -1] = [+1, +1, +1, +1] = Id \\
 f^2 = [+1, -1, -1, +1](\equiv)[+1, -1, -1, +1] = Id \\
 g^2 = [+1, -1, +1, -1](\equiv)[+1, -1, +1, -1] = Id
 \end{array} \tag{16}$$

Here, we indicate that component-wise multiplication is brought forth by logical equivalence of sequences. Also we have

$$\begin{array}{l}
 e_3 e_3 = \tau f \tau f = f \tau \tau f = f Id f = f f Id = Id Id = Id \\
 e_4 e_4 = f \varphi f \varphi = -\varphi f f \varphi = -\varphi Id \varphi = -\varphi \varphi Id = -Id Id = -Id
 \end{array} \tag{17}$$

We summarise the first and most important result: $e_1^2 = e_2^2 = e_3^2 = Id, e_4^2 = -Id$. We have also verified the (anti)commutation relations for Clifford algebra $Cl_{3,1}$. As demanded by traditional mathematical physics, we could represent the iterants e, f , as well as the transpositions by 4 x 4 matrices

$$e := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \quad f := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad \varphi := \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \quad \sigma \stackrel{\text{def}}{=} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

What does this mean? The matrices e, f correspond (1) with the iterants or polarity strings, briefly, $[+ + - -]$, $[+ - - +]$. Logically these represent (2) two different atomic statements A and B in Boolean logic, also symbolised by (3) the icons \sqcap and \sqcup , which correspond with two idempotents in the Minkowski algebra, namely (4) $\sqcap \simeq f_1 + f_2$ and $\sqcup \simeq f_1 + f_4$. These two, simplest logic connectives, together with two transpositions of locations of characters, namely $\varphi := (1\ 3)(2\ 4)$, exchanging location 1 with 3 and 2 with 4; and $\sigma := (1\ 2)(3\ 4)$, exchanging location 1 with 2 and 3 with 4,

²⁴ See Schmeikal, "Four Forms Make a Universe," Table 17.

generate the basis not only of Minkowski space, but also of its 16-dimensional geometric algebra. Is not this a surprise? Two statements and two transpositions of characters in a linear fourfold array give rise to the basis of space-time geometry. We could represent this by matrices. The Minkowski space would thus be given by familiar 4 x 4 matrices:

$$e_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \quad e_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \quad e_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; \quad e_4 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

But, the more important result is that any dynamic process in space-time algebra can be processed line after line by manipulation of iterants with the circuits of autonomic intelligence.

Towards Line Processing of $SU(3) \subset CI(3, 1)$

Using (15) and the imaginary unit i for convenience, we can compute a representation of the $SU(3)$ that can be written in one line instead of as matrices, in terms of transpositions φ, σ, τ combined with fourfold linear arrays, which means that the intellector needs two transposing swap gates, τ being immediately recognised as the present palindromic operation (19)

$$\begin{aligned} T_1 &= \frac{1}{4}(Id - f) = \frac{1}{2}\tau[0,1,1,0] & T_2 &= \frac{i}{4}\tau(e - g) = \frac{i}{2}\tau[0,1, -1,0] \\ T_3 &= \frac{1}{4}(e - g) = \frac{1}{2}[0,1, -1,0] & T_4 &= \frac{1}{4}\varphi(Id - g) = \frac{1}{2}\varphi[0,1,0,1] \\ T_5 &= \frac{i}{4}\varphi(e - f) = \frac{i}{2}\varphi[0,1,0, -1] & T_6 &= \frac{1}{4}\sigma(Id - e) = \frac{1}{2}\sigma[0,0,1,1] \\ T_7 &= \frac{i}{4}\sigma(g - f) = \frac{i}{2}\sigma[0,0,1, -1] & T_8 &= \frac{1}{4\sqrt{3}}(e - 2f + g) = \frac{1}{2\sqrt{3}}[0,1,1, -2] \end{aligned}$$

These expressions resemble up to a factor $\frac{1}{2}$ the Gell-Mann matrices. In the corresponding matrix representation, we would consider zero in the first line and first column. What we need for line processing is the possibility of carrying out transpositions φ, σ, τ of four characters in line-arrays. On this basis, we can calculate $t-, u-$ and $v-$ spin. For example, let us calculate the isospin shift operators:

$$T_{\pm} = \frac{1}{\sqrt{2}}(T_1 \pm iT_2) = \frac{1}{2\sqrt{2}}(\tau[0,1,1,0] \mp \tau[0,1, -1,0]) \rightarrow T_+ = \frac{1}{\sqrt{2}}\tau[0,0,1,0] \quad (20)$$

and $T_- = \frac{1}{\sqrt{2}}\tau[0,1,0,0]$. Now verify the commutator equations for shift operators, first the product

$$\begin{aligned} T_3 T_+ &= \frac{1}{2\sqrt{2}}[0,1, -1,0]\tau[0,0,1,0] = \frac{1}{2\sqrt{2}}\tau[0, -1,1,0][0,0,1,0] = \frac{1}{2\sqrt{2}}\tau[0,0,1,0], \text{ next} \\ T_+ T_3 &= \frac{1}{2\sqrt{2}}\tau[0,0,1,0][0,1, -1,0] = \frac{1}{2\sqrt{2}}\tau[0,0, -1,0]; \text{ therefore, we obtain the commutator} \\ [[T_3, T_+]] &= T_3 T_+ - T_+ T_3 = \frac{1}{\sqrt{2}}\tau[0,0,1,0] = T_+ \text{ and likewise we get } [[T_3, T_-]] = -T_- \quad (21) \end{aligned}$$

In this representation, the f_1 is a fixed lepton, and f_2, f_3, f_4 are quarks. We have

$$\begin{aligned} T_3 f_2 &= \frac{1}{2} [0, 1, -1, 0][0, 1, 0, 0] = \frac{1}{2} [0, 1, 0, 0] = \frac{1}{2} f_2 \\ T_8 f_2 &= \frac{1}{2\sqrt{3}} [0, 1, 1, -2][0, 1, 0, 0] = \frac{1}{2\sqrt{3}} [0, 1, 0, 0] = \frac{1}{2\sqrt{3}} f_2 \end{aligned} \quad (22)$$

The eigenvector $f_2 = [0, 1, 0, 0]$ corresponds to a state $|\mu\rangle$, where $\mu = (\mu_1, \mu_2) = \left(+\frac{1}{2}, +\frac{1}{2\sqrt{3}}\right)$ is distinguished by its eigenvalues under the operators T_3, T_8 of the Cartan subalgebra of $SU(3, \mathbb{C})$. We also have that

$$\begin{aligned} T_3 f_3 &= \frac{1}{2} [0, 1, -1, 0][0, 0, 1, 0] = -\frac{1}{2} [0, 0, 1, 0] = -\frac{1}{2} f_3 \\ T_8 f_3 &= \frac{1}{2\sqrt{3}} [0, 1, 1, -2][0, 0, 1, 0] = \frac{1}{2\sqrt{3}} [0, 0, 1, 0] = \frac{1}{2\sqrt{3}} f_3, \text{ corresponding to the weight vector} \\ \mu' &= \left(-\frac{1}{2}, +\frac{1}{2\sqrt{3}}\right), \text{ and finally} \end{aligned} \quad (23)$$

$$\begin{aligned} T_3 f_4 &= \frac{1}{2} [0, 1, -1, 0][0, 0, 0, 1] = 0 \\ T_8 f_4 &= \frac{1}{2\sqrt{3}} [0, 1, 1, -2][0, 0, 0, 1] = \frac{1}{2\sqrt{3}} [0, 0, 0, -2] = -\frac{1}{\sqrt{3}} f_4 \text{ for the weight vector} \\ \mu'' &= \left(0, +\frac{1}{\sqrt{3}}\right) \end{aligned}$$

Primitive idempotents f_2, f_3, f_4 can thus be identified with quark-states $|u\rangle, |d\rangle, |s\rangle$. Notice, that the Cartan algebra $\{T_3, T_8\}$ is a subalgebra of the Cartan algebra of the rank 3 symmetric unitary group $SU(4) \subset \mathbb{C} \otimes Cl_{3,1}$,²⁵ which is given by the three commuting multivectors $\{e_1, e_{24}, e_{124}\}$, or what we have abbreviated by $\{e, f, g\}$.

Words of LICO

When discussing dialectical machine vision, Isaacson performs a turnover from BIP to DIP phenomenology.²⁶ We are confronted with neural circuits, constituted by three types of neurons, namely, *type C – central*, *type P – peripheral*, and *type H – horizontal*. A CA DIP-cell is represented by a C-neuron surrounded by eight P-neurons in a regular arrangement.²⁷ In my paper “On Consciousness & Consciousness Logging Off Consciousness,”²⁸ I tried to go back in time to see what happened. DIP had begun with a 2D, 256-state Moore-neighbourhood cellular automaton. This was realised by a highly interacting network of BIPs. Inputs to DIP were some digitised images,²⁹ given by silhouettes of objects, so-called *retinels*, embedded in some ground, both represented by pixels with different grey values. The CA operated on the input image by carrying out an 8-way comparison of each pixel with its eight neighbours, giving a difference or no difference. Each single, 8-way comparison thus yielded one value out of $2^8 = 256$ possible ones. Each retinel was then written down by words in an ideographic *alphabet of the visual cortex* with 256 letter-shapes, each of which represented one of the

²⁵ The connection between $Cl_{3,1}$, $SU(4, \mathbb{C})$ and $SL(4, \mathbb{C})$ is described by Lie brackets in Chapter 2 of Bernd Schmeikal, *Primordial Space – Pointfree Space and Logic Case* (New York: Nova Science, 2012).

²⁶ Isaacson, “Dialectical Machine Vision,” 35.

²⁷ *Ibid.*, Figure 13.

²⁸ Bernd Schmeikal, “On Consciousness & Consciousness Logging Off Consciousness,” www.researchgate.net/publication/289335467_On_Consciousness, January 2016, 11-15.

²⁹ See Isaacson, “Dialectical Machine Vision,” Figures 10 to 12.

possible relationships. These fonts resemble the 16 letters of the logic alphabet LICO. In my view, Isaacson’s resistance to describing DIP in neural terms, has a much more serious reason than those social entanglements affiliated with McCulloch, Pitts, Foerster and the Perceptron,³⁰ namely, the notable features of the retinoid neuronal system of consciousness follow a deeper template that is more fundamental than neural nets. Its realization by the process of nature is at least as simple as a CA of the type DIP. The ground template is provided by the process of space-time itself. Primordial space provides rules for the formation of elementary particles, atoms, chemical elements, biomolecules, and genetic code.

In my article “Free Linear Iconic Calculus - AlgLog Part 1,” I showed how the shapes of the iconic letters can be understood in two ways, namely (1) by studying the truth tables of the corresponding Boolean connectives, or (2) by simply representing each primitive idempotent of the $Cl_{3,1}$ by a bar in a little square. Each icon has an algebraic expression in terms of four primitive idempotents f_1, f_2, f_3, f_4 (see Column 4 of Table 2)³¹ of the Clifford algebra $Cl_{3,1}$ which span the colourspace ch_1 , and likewise as multivector in this linear commutative vector space (see Columns 5 and 7). Every icon can also be obtained by superimposing four icons, which represent the basis of this space.

Table 2: Correspondences in Algebra Structures for Logic Icons

Nr. icon	Boole	LICO letter	Idempotents f in $Cl_{3,1}$	f Rep in colourspace $ch_1 \subset Cl_{3,1}$	Polarity string	Deformed Polarity string Rep in colourspace ch_1
J_{01}	$A \wedge \neg A$		0	0	[- - - -]	$-Id$
J_{02}	$A \wedge B$		f_1	$\frac{1}{4}(Id + e_1 + e_{24} + e_{124})$	[+ - - -]	$\frac{1}{2}(-Id + e_1 + e_{24} + e_{124})$
J_{03}	$A \wedge \neg B$		f_2	$\frac{1}{4}(Id - e_1 + e_{24} - e_{124})$	[- + - -]	$\frac{1}{2}(-Id + e_1 - e_{24} - e_{124})$
J_{04}	$\neg A \wedge \neg B$		f_3	$\frac{1}{4}(Id - e_1 - e_{24} + e_{124})$	[- - + -]	$\frac{1}{2}(-Id - e_1 - e_{24} + e_{124})$
J_{05}	$\neg A \wedge B$		f_4	$\frac{1}{4}(Id + e_1 - e_{24} - e_{124})$	[- - - +]	$\frac{1}{2}(-Id - e_1 + e_{24} - e_{124})$
J_{06}	A		$f_1 + f_2$	$\frac{1}{2}(Id + e_1)$	[+ + - -]	e_1
J_{07}	$\neg A$		$f_3 + f_4$	$\frac{1}{2}(Id - e_1)$	[- - + +]	$-e_1$
J_{08}	A ≡ B		$f_1 + f_3$	$\frac{1}{2}(Id + e_{124})$	[+ - + -]	e_{124}
J_{09}	A ≠ B		$f_2 + f_4$	$\frac{1}{2}(Id - e_{124})$	[- + - +]	$-e_{124}$
J_{10}	B		$f_1 + f_4$	$\frac{1}{2}(Id + e_{24})$	[+ - - +]	e_{24}
J_{11}	$\neg B$		$f_2 + f_3$	$\frac{1}{2}(Id - e_{24})$	[- + + -]	$-e_{24}$
J_{12}	$A \vee B$		$f_1 + f_2 + f_4$	$\frac{1}{4}(3Id + e_1 + e_{24} - e_{124})$	[+ + - +]	$\frac{1}{2}(Id + e_1 + e_{24} - e_{124})$
J_{13}	$\neg A \vee B$		$f_1 + f_3 + f_4$	$\frac{1}{4}(3Id - e_1 + e_{24} - e_{124})$	[+ - + +]	$\frac{1}{2}(Id - e_1 + e_{24} + e_{124})$

³⁰ Warren S. McCulloch and Walter Pitts, “A Logical Calculus of the Ideas Immanent in Nervous Activity,” *Bulletin of Mathematical Biology* 5, no. 4 (1943): 115-33; Heinz von Foerster, *Das Gedächtnis: Eine Quantenphysikalische Untersuchung* (Vienna: Franz Deuticke, 1948).

³¹ Bernd Schmeikal, “Algebra of Quantum Logic,” in *Clifford Algebras and their Application in Mathematical Physics*, ed. R. Ablamowicz and B. Fauser, (Boston: Birkhäuser, 2000), 219-41.

J_{14}	$A \vee \neg B$	\sqsupset	$f_1 + f_2 + f_3$	$\frac{1}{4}(3Id + e_1 - e_{24} + e_{124})$	$[+ + + -]$	$\frac{1}{2}(Id + e_1 - e_{24} + e_{124})$
J_{15}	$\neg A \vee \neg B$	\sqsubset	$f_2 + f_3 + f_4$	$\frac{1}{4}(3Id - e_1 - e_{24} - e_{124})$	$[- + + +]$	$\frac{1}{2}(Id - e_1 - e_{24} - e_{124})$
J_{16}	$A \vee \neg A$	\square	$\sum_i^4 f_i = Id$	Id	$[+ + + +]$	$+Id$

We can write

$$ch_1 = span_{\mathbb{R}}\{Id, A, B, \equiv\} = span_{\mathbb{R}}\{\square, \sqsupset, \sqsubset, \square\} \simeq span_{\mathbb{R}}\{Id, e_1, e_{24}, e_{124}\}$$

For example, we can obtain the logic adjunction $A \vee B$ by superimposing the generating icons

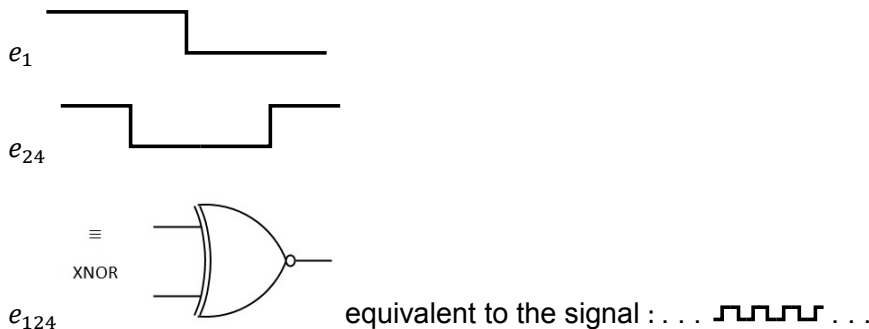
$$A \vee B \simeq \square = \frac{1}{2}(\square + \sqsupset + \sqsubset - \square)$$

That is, we get three minus one on the upper bar, which, divided by two, gives one on the upper bar; one minus one on the lower bar, giving zero; two on the left bar divided by two, giving one bar on the left; and two divided by two, giving one bar on the right. Hence, the icon looks like \square . In analogous way, any of the 16 icons can be obtained from the four $\square, \sqsupset, \sqsubset, \square$. What is interesting is that these four can be represented by binary sequences or polarised strings or by logic circuits. The most important representation seems to be given by the XNOR gate, symbolised by the identifying connective \equiv , and one of the sequences, say $[+ + - -]$ and the two swap gates. There is a special beauty in such a design, as the logic equivalence comes in as an element of both the carrier set and the binary operation. Hence, when we multiply A with B we actually have the expression

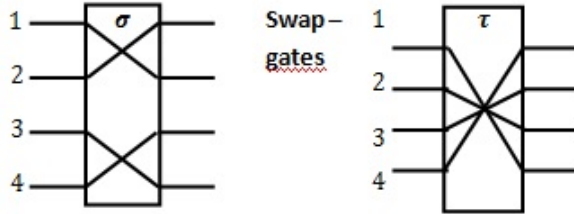
$$A B = [+ +, - -, - +, - +] [+ +, - -, - +] = [+ +, - -, - +, - +] representing the identity $A \equiv B$.$$

So we can span this invariant subspace ch_1 by one of two signals A, B and the identity machine:

1) generating base units



We need two swap gates to have the whole Clifford algebra of space-time. It is not a problem to imagine how these elements could be realised by neurons.



I would suggest that we design space-time the way we do it in Clifford algebra, because our brain functions in a way that prompts such a mathematical design. Surprisingly, Isaacson's BIP was indeed built up with the aid of such binary signals and by arrays of parallel XNOR gates, which is essential for the emergence of a Minkowskian space-time algebra. But I am missing one swap-gate that is necessary to iterate from the Cartan subalgebras. The BIP brings forth important features of the commutative subalgebras of the angular 4-momentum subspaces of HEPHy. Interpreted as articulations of neighbourhood, the icons of LICO describe a topological procedure operating on strings. We can apply LICO to LICO words. Then we obtain Hegelian cycles of recurring patterns of idempotent locations that may be interpreted as events in angular 4-momentum space, as dynamic processing of $SU(4)$ - and $SU(3)$ -multiplets (see Figure 5).

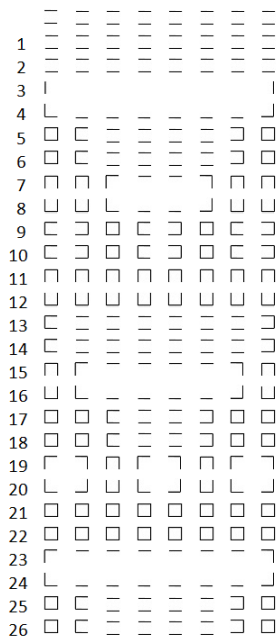


Figure 5: Recurring patterns of strings of idempotent locations.

This periodic pattern can also be generated by starting off from the second line of three empty neighbourhoods. Isaacson's discovery of how a simple cellular automaton unexpectedly encoded the baryon octet of elementary particle physics is not just a happy coincidence, but it is rooted in a deep archetypal connection between geometry and logic.³² We do not yet know where the journey goes, but it seems that the rigor completely works out.

Recursive Distinctioning and Antecursive Conflation

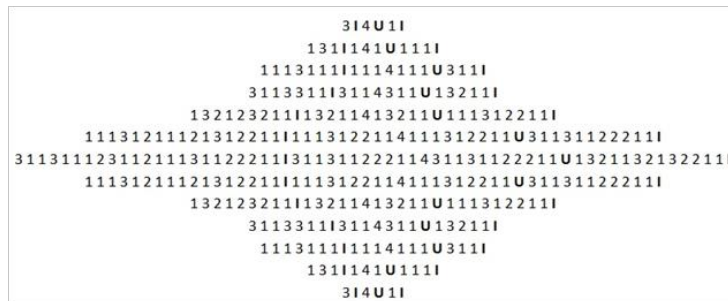
Isaacson and Kauffman have written a paper about what they call RD.³³ They have also written an advance statement as a letter in the *Journal of Space Philosophy*, to which I now refer.³⁴ This whole undertaking, which seems to carry on the torch of Hegelian dialectics, that is, limit cycles seen by a theory of dialectic cyclic development, may be fraught with problems philosophically, but seems serious mathematical business. It is also based on Kauffman's own work on recursion and distinction in cybernetics and his peculiar care for the investigations of George S. Brown into the *laws of form*.³⁵ Personally, I was not overmuch delighted by this darting off, since the two experts in *Hegelian Ansatz* seem to have neglected exactly one half of the dialectics and one half of the story of evolution, which has much of the quality of a fairy tale, anyway. In order to see the whole, I made a hotfoot invention of the dual process, that is, the *ante-cursive conflation*. To see what that is, imagine a UFO coming in from the horizon of your world. It looks like this:

³² Joel D. Isaacson, *Steganogramic Representation of the Baryon Octet in Cellular Automata*, (St. Louis, MO: IMI Corporation, 2015), www.iss.org/2001meet/2001paper/stegano.pdf (accessed December 8, 2015).

³³ *Journal of Space Philosophy* 5, no. 1 (Spring 2016): 9-63.

³⁴ Joel D. Isaacson and Louis H. Kauffman, "Recursive Distinguishing," *Journal of Space Philosophy* 4, no. 1 (2015): 23-27.

³⁵ Louis K. Kauffman, *Map Reformulation* (London: Princelet Editions, 1986).



As it approaches the egoistic centre of your world, its ideographic shape becomes a readable message, readable even in the familiar sense, from left to right, from top to bottom. But you cannot read it, because some rules that would provide the meaning are missing. Fortunately, a kid comes in and informs you that, at present, the **I** means 'I' as usual, that is, ego, and the **U** means you. 3 **I** should denote a written sequence of three I's, that is, **III**, whereas 4 **U** would stand for the sequence **UUUU**, that would be all. So you write down the 8-letter word

IIUUUUI

Being a philosopher, someone who likes wisdom, you can see the meaning of this word: It all has to begin with an invisible sentence, namely "I referring to m I self and I am referring to You and You are referring to Yourself and You are referring to me." So the top of the UFO "3 I 4 U 1 I" is a description of the line **IIUUUUI**. The next line should be a description of line 3 I 4 U 1 I. Reading character by character, we see that we have one '3', that is, 1 3, further one **I**, that is, 1 I, further one '4', that is, 1 4, next 1 **U**, and so on. Altogether, we get 1 3 1 I 1 4 1 U 1 1 I. This provides the rule for the RD. Applying the rule many times, we obtain line after line following on from Invisible Line 1: I referring to m I self and I referring to You and you referring to Yourself and You referring to me.

Invisible Line 2: I I I U U U U I

Line 3: 3 I 4 U 1 I

Line 4: 1 3 1 I 1 4 1 U 1 1 1 I (Line 4 describing Line 3)

Line 5: 1 1 1 3 1 1 1 I 1 1 1 4 1 1 1 U 3 1 1 I

Line 6: 3 1 1 3 3 1 1 I 3 1 1 4 3 1 1 U 1 3 2 1 1 I

Line 7: word₁I word₂You word₃I (describing Line 6)

Recursive
D
i
s
t
i
n
g
u
ishing

It turns out that the domain of interpersonal experience (I and You) is transformed by mathematical self-reference/reflexive linguistic domain into a wilderness of numbers by which I and U are isolated from each other. There is some segregating demon in the mathematical detail. It seems that a wilderness of numbers is not necessarily essentially different from a wilderness of letters, is not essentially different from a wilderness of words, is not of sentences of threads and so on ad infinitum.

But now do the reverse! Now you have to be attentive for pairs of characters: 3 1 means '111'

For example, begin with: 3 1 1 3 3 1 1 I 3 1 1 4 3 1 1 U 1 3 2 1 1 I

Line 6: 3 1 1 3 3 1 1 I 3 1 1 4 3 1 1 U 1 3 2 1 1 I

Make 5: 1 1 1 3 1 1 1 I 1 1 1 4 1 1 1 U 3 1 1 I

Make 4: 1 3 1 I 1 4 1 U 1 1 1 I

Make 3: 3 I 4 U 1 I

Make 2: I I I U U U U I

Antecursive
C
o
n
f
l
a
t
i
o
n

By doing antecursive conflation, we get rid of the separating mass of numbers, and we are back at U and I. In this way, we obtain the lower half of the UFO converging towards our six letter word 3 I 4 U 1 I.

3 1 1 3 1 1 1 2 3 1 1 2 1 1 1 3 1 1 2 2 2 1 1 I 3 1 1 3 1 1 2 2 2 1 1 4 3 1 1 3 1 1 2 2 2 1 1 U 1 3 2 1 1 3 2 1 3 2 2 1 1 I
 1 1 1 3 1 2 1 1 1 2 1 3 1 2 2 1 1 I 1 1 1 3 1 2 2 1 1 4 1 1 1 3 1 2 2 1 1 U 3 1 1 3 1 1 2 2 2 1 1 I
 1 3 2 1 2 3 2 1 1 I 1 3 2 1 1 4 1 3 2 1 1 U 1 1 1 3 1 2 2 1 1 I
 3 1 1 3 3 1 1 I 3 1 1 4 3 1 1 U 1 3 2 1 1 I
 1 1 1 3 1 1 1 I 1 1 4 1 1 1 U 3 1 1 I
 1 3 1 1 4 1 U 1 1 1 I
 3 I 4 U 1 I

What is important is to see the difference in **being aware for one definite character** while describing it by RD, and **being attentive for a relation**, that is, two characters, **while carrying out an antecursive conflation**. Future work will clarify this final issue of analysis.

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About the Author: Bernd Anton Schmeikal, born May 15, 1946, a retired freelancer in research and development, qualified in Sociology with a treatise about cultural time reversal. He is a real maverick, still believing that social life can be based on openness and honesty. As a PhD philosopher from Vienna, with a typical mathematical physics background, he entered the trace analysis group of the UA1 Experiment at CERN, under the leadership of Walter Thirring, in 1965. This was in the foundation phase of the Institute for High Energy Physics (HEPhy) at the Austrian Academy of Science. He has always been busy solving fundamental problems concerning the unity of matter and space-time, the origin of the HEPhy standard model, and the phenomenology of relativistic quantum mechanics. In the Sociology Department of the Institute for Advanced Studies (HIS Vienna), he helped James Samuel Coleman to conceive his mathematics of collective action as a cybernetic system, and gave the process of *internalization of collective values* an exact shape. He implemented many transdisciplinary research projects for governmental and non-governmental organizations, universities and non-university institutions, and several times introduced new views and methods. He founded an international work stream that, for the first time, worked under the name of the Biofield Laboratory (BILAB). Although close to fringe science and electromedicine, the work of BILAB had a considerable similarity with the Biological Computer Laboratory run earlier by Heinz von Foerster. Lately, he has applied Foerster's idea of a universal relevance of hyperbolic distributions (Zipf's law) in social science to the labour market. This signifies a last contribution to the research program of the Wiener Institute for Social Science Documentation and Methodology (WISDOM) under the sponsorship of the Austrian Federal Presidential Candidate Rudolf Hundstorfer. He is convinced that a unity of science and culture can be achieved, but that this demands more than one Einstein. Consequently, he sought cooperation with Louis Kauffman and Joel Isaacson.



Editors' Notes: Dr. Bernd Schmeikal's review and evaluation of Joel Isaacson and Louis Kauffman's RD research and paper, published in this Journal, is a very valuable contribution to this forefront science investigation of *Nature's Cosmic Intelligence*. Dr. Schmeikal, University of Vienna Professor in mathematics, linguistics, and physics is one of the world's distinguished scholars for this special field of universe autonomous intelligence. He begins his abstract with the statement: "This paper investigates a universal creative system," and ends it with "That is to say, our universe may be a representation of Isaacson's system, and entertainingly, with his US Patent specification 4,286,330, it seems he has patented creation." **Bob Krone and Gordon Arthur.**

Journal of Space Philosophy (JSP) Board of Editors

Kepler Space Institute (KSI) is honored to have 42 of the world's Space community professionals as members of the Board of Editors for the *Journal of Space Philosophy*.

Dr. Elliott Maynard, our *Journal of Space Philosophy* Board of Editors colleague, has beautifully stated both the purpose and the style for our peer reviews:

This is such a hi-caliber group of leading-edge thinkers and supercharged individuals, it should be natural for each of us to wish to provide a supportive and synergistic environment for the others. I have also learned always to have someone else proof read any material I write, as I have discovered that the brain tends not to "see" my own simple mistakes. Ergo, within the new Kepler context I feel editors should be there to support our writers in the most creative and positive ways possible. (e-mail to Bob Krone, March 23, 2013)

The purposes of peer reviews of article submissions to the *Journal of Space Philosophy* are: (1) to determine the relevance to the Vision and Goals of KSI; (2) to help the author(s) improve the article in substance and style or recommend references; and (3) to provide publication recommendations to the Editor-in-Chief.

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BEN-JACOB, Eshel, PhD, Former President of Israel Physical Society; Founder Science of Bacterial Intelligence. Tel Aviv University. We grieve the passing of Dr. Ben-Jacob in 2015.

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5.



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For Bio Info: See Issue 1, no. 1, Article 11.









10.











FITZPATRICK, Susan Beaman, DBA, Vice Chairman, Oak Family Advisors, LLC based in Chicago. She earned her DBA with the University of South Australia in Zurich Switzerland, where she studied under the supervision of Dr. Bob Krone. She is an international health expert specializing in health risk management. She has consulted with governments, public and private providers, and within health systems projects sponsored by the World Bank, World Health Organisation, and the UK’s National Health Service. Susan’s research interests include management capacity development and the implementation of complex innovations and programs. She has been a keynote speaker at industry symposiums and professional organizations such as the National Risk Manager’s Association, Excess Surplus Lines Claims Association, American Hospital Association, American Bar Association, and State Chambers of Commerce. Kepler Space Institute is proud to have her in the *Journal of Space Philosophy* Board of Editors.

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“The greatest use of a life is to spend it for something positive that outlasts it.” Dr. Max T. Krone, Dean, Institute of the Arts, University of Southern California and Founder, Idyllwild School of Music and the Arts, 1950

